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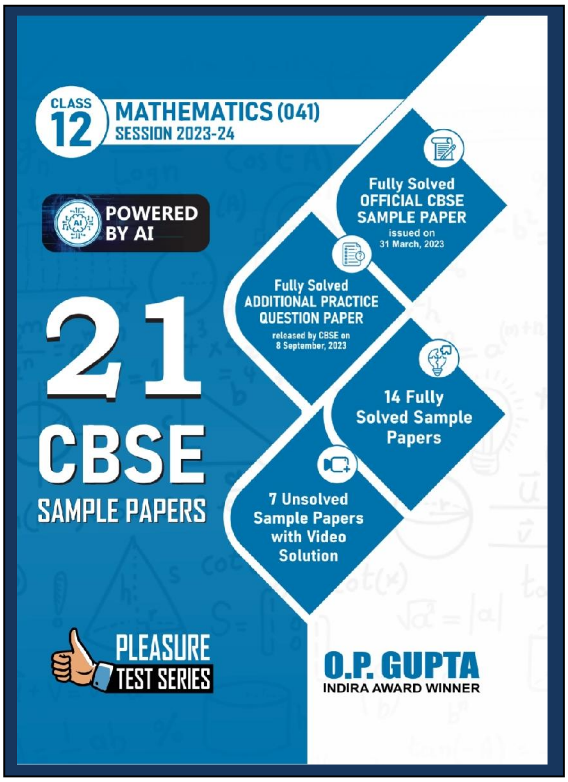

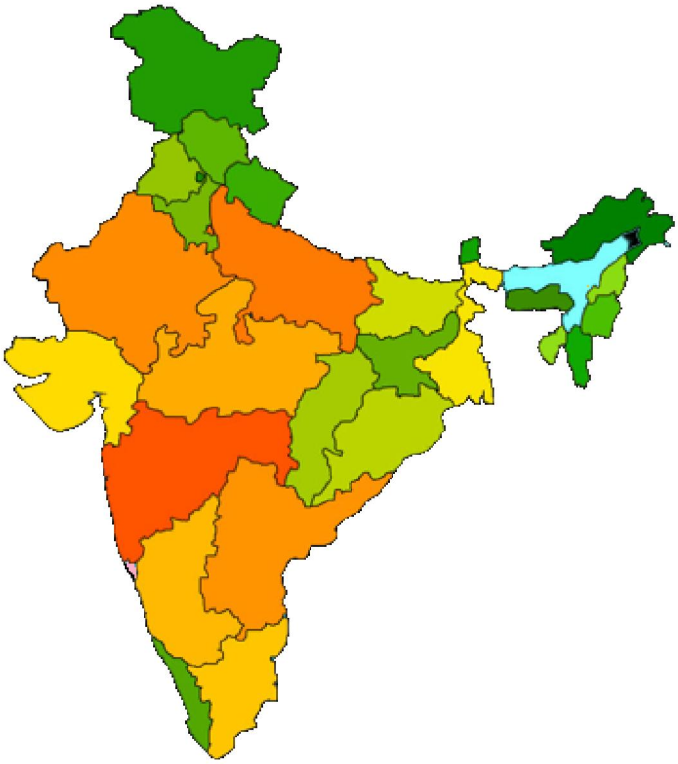

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Some tips for excelling well in the CBSE 2024 Board Exams

1. Understand the Syllabus: Ensure you are familiar with the entire CBSE Class XII Maths syllabus. Focus on the weightage of each unit to prioritize your preparation.

2. Practice Regularly: Mathematics is about practice. Solve a variety of problems from different exercises of the chapters regularly to enhance your problem-solving skills.

3. Master the Basics: Make sure you have a strong foundation in basic concepts. Understanding fundamentals will help you tackle complex problems with ease.

4. Time Management: Practice solving problems within the stipulated time. Develop a strategy to manage your time during the exam, allocating sufficient time to each section.

5. NCERT Textbook: Stick to the NCERT textbooks for Class XII Maths. CBSE exams are primarily based on it, and it covers **almost all** (*not all*) the essential concepts.

6. Previous Year Papers (PYQs): Solve previous years question papers to understand the exam pattern and types of questions that may be asked. This will also help you manage your time effectively during the exam.

7. Make Notes: Prepare concise notes while studying. These notes can serve as a quick revision tool before the exam.

8. Focus on Weak Areas: Identify your weak areas and spend extra time on them. This will help you improve your overall performance.

9. Use Diagrams and Formulas: For the geometry problems, draw neat diagrams. Focus to memorize all the important formulas and practice their application.

10. Stay Calm: During the exam, if you encounter a difficult question, remain calm. Move on to the next one and come back to it later if time permits.

11. Revise Thoroughly: In the days leading up to the exam, focus on revision. Revise all the important formulas, theorems, and concepts.

12. Clarify Doubts: If you have any doubts, clarify them with your teacher or peers (*you may also post your doubts in our WhatsApp / Telegram groups*). It's essential to have a clear understanding of all topics.

Remember, **consistent and focused preparation** is the key to performing well in your CBSE XII Maths examination.

TABLE OF CONTENTS

Syllabus
CBSE EXAMS (2023-24)
Class XII ▪ Maths (041)

One Paper (Theory)
Time: 180 Minutes Max Marks: 80

No.	UNITS	MARKS
I	Relations & Functions	08
II	Algebra	10
III	Calculus	35
IV	Vectors & 3 D Geometry	14
V	Linear Programming	05
VI	Probability	08
	Total	**80**

CBSE S.Q.P. (2023-24) • MATHEMATICS (041) • XII

Prepared by ***O.P. GUPTA*** ***Indira Award Winner***

Note *: This Bifurcation of Questions is based on Sample Question Paper issued by CBSE, for the Board Examinations 2024.*

Chapters	**Section A** **(1 mark)** ***MCQ type***	**Section B** **(2 marks)** ***VSA type***	**Section C** **(3 marks)** ***SA type***	**Section D** **(5 marks)** ***LA type***	**Section E** **(Case Study)** **(4 marks)** ***Subjective type***	**Marks for each unit**
Relations & Functions	Q20 **(A.R.)**			Q33*		**8**
Inverse Trig. Functions		Q21*				
Matrices & Determinants	Q01, 02, 03, 10, 13			Q34		**10**
Continuity & Differentiability	Q04, 17		Q31			**35**
Applications of Derivatives	Q19 **(A.R.)**	Q22, Q23*, Q25			Q38 (*with 2 parts*)	
Integrals	Q09	Q24	Q26, Q28*			
Application of Integrals				Q32		
Differential Equations	Q06, 15		Q29*			
Vector Algebra	Q08, 12, 16				Q37* (*with 3 parts*)	**14**
3 Dimensional Geometry	Q05, 18			Q35*		
Linear Programming	Q07, 11		Q30*			**5**
Probability	Q14		Q27		Q36* (*with 3 parts*)	**8**
Total Marks	**20 Marks**	**10 Marks**	**18 Marks**	**20 Marks**	**12 Marks**	**80 Marks**

**** Internal choices given for the concerned questions based on the mentioned topics / units.***

SAMPLE PAPER

issued by CBSE for Board Exams (2023-24)

Mathematics (041) - Class 12

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (4 **marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

SECTION A

*(Question numbers 01 to 20 carry **1 mark** each.)*

*Followings are **multiple choice questions**. Select the correct option in each one of them.*

01. If $A=[a_{ij}]$ is a square matrix of order 2 such that $a_{ij}=\begin{cases}1, \text{ when } & i\neq j\\ 0, \text{ when } & i=j\end{cases}$, then A^2 is

(a) $\begin{bmatrix}1&0\\1&0\end{bmatrix}$ (b) $\begin{bmatrix}1&1\\0&0\end{bmatrix}$ (c) $\begin{bmatrix}1&1\\1&0\end{bmatrix}$ (d) $\begin{bmatrix}1&0\\0&1\end{bmatrix}$

02. If A and B are invertible square matrices of the same order, then which of the following is **not** correct?

(a) $\text{adj.}A=|A|A^{-1}$ (b) $\det.(A)^{-1}=[\det.(A)]^{-1}$

(c) $(AB)^{-1}=B^{-1}A^{-1}$ (d) $(A+B)^{-1}=B^{-1}+A^{-1}$

03. If the area of the triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 Sq. units, then the value/s of k will be

(a) 9 (b) ±3 (c) –9 (d) 6

04. If $f(x)=\begin{cases}\dfrac{kx}{|x|}, \text{ if } x<0\\ 3, \text{ if } x\geq 0\end{cases}$ is continuous at $x=0$, then the value of k is

(a) –3 (b) 0 (c) 3 (d) any real number

05. The lines represented by $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$ and $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(6\hat{i}+9\hat{j}-18\hat{k})$; (where λ and μ are scalars) are

(a) coincident (b) skew (c) intersecting (d) parallel

06. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}\right)^2$ is

(a) 4 (b) $\frac{3}{2}$ (c) 2 (d) not defined

07. The corner points of the bounded feasible region determined by a system of linear constraints are $(0, 3)$, $(1, 1)$ and $(3, 0)$. Let $Z=px+qy$, where $p, q>0$. The condition on p and q so that the minimum of Z occurs at $(3, 0)$ and $(1, 1)$ is

(a) $p=2q$ (b) $p=\frac{q}{2}$ (c) $p=3q$ (d) $p=q$

08. $ABCD$ is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}=$

(a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BD}$ (d) $2\overrightarrow{AD}$

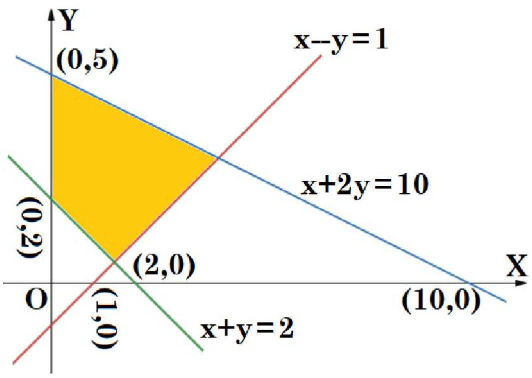
09. For any integer n, the value of $\int_0^{\pi} e^{\sin^2 x}\cos^3(2n+1)x\,dx$ is

(a) -1 (b) 0 (c) 1 (d) 2

10. The value of $|A|$, if $A=\begin{bmatrix}0 & 2x-1 & \sqrt{x}\\ 1-2x & 0 & 2\sqrt{x}\\ -\sqrt{x} & -2\sqrt{x} & 0\end{bmatrix}$, where $x\in R^+$, is

(a) $(2x+1)^2$ (b) 0 (c) $(2x+1)^3$ (d) None of these

11. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.



Which of the following is **not** a constraint to the given Linear Programming Problem?

(a) $x+y\ge 2$
(b) $x+2y\le 10$
(c) $x-y\ge 1$
(d) $x-y\le 1$

12. If $\vec{a}=4\hat{i}+6\hat{j}$ and $\vec{b}=3\hat{j}+4\hat{k}$, then the vector form of the component of $\vec{a}$ along $\vec{b}$ is

(a) $\frac{18}{5}(3\hat{i}+4\hat{k})$ (b) $\frac{18}{25}(3\hat{j}+4\hat{k})$ (c) $\frac{18}{5}(3\hat{i}+4\hat{k})$ (d) $\frac{18}{25}(4\hat{i}+6\hat{j})$

13. Given that A is a square matrix of order 3 and $|A|=-2$, then $|\text{adj.}(2A)|$ is equal to

(a) -2^6 (b) 4 (c) -2^8 (d) 2^8

14. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

15. The general solution of the differential equation $ydx-xdy=0$; (given $x, y>0$), is of the form

(a) $xy=c$ (b) $x=cy^2$ (c) $y=cx$ (d) $y=cx^2$

16. The value of λ, for which two vectors $2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\lambda\hat{j}+\hat{k}$ are perpendicular is
(a) 2 (b) 4 (c) 6 (d) 8

17. The set of all points where the function $f(x)=x+|x|$ is differentiable, is
(a) $(0,\infty)$ (b) $(-\infty,0)$ (c) $(-\infty,0)\cup(0,\infty)$ (d) $(-\infty,\infty)$

18. If the direction cosines of a line are $<\frac{1}{c},\frac{1}{c},\frac{1}{c}>$ then
(a) $0<c<1$ (b) $c>2$ (c) $c=\pm\sqrt{2}$ (d) $c=\pm\sqrt{3}$

*Followings are **Assertion-Reason based questions**.*
In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).
Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}\left(f(x)\right)=(x-1)^3(x-3)^2$.
Assertion (A) : $f(x)$ has a minimum at $x=1$.
Reason (R) : When $\frac{d}{dx}\left(f(x)\right)<0,\ \forall x\in(a-h,a)$ and $\frac{d}{dx}\left(f(x)\right)>0,\ \forall x\in(a,a+h)$; where 'h' is an infinitesimally small positive quantity, then $f(x)$ has a minimum at $x=a$, provided $f(x)$ is continuous at $x=a$.

20. **Assertion (A) :** The relation $f:\{1,2,3,4\}\to\{x,y,z,p\}$ defined by $f=\{(1,x),(2,y),(3,z)\}$ is a bijective function.
Reason (R) : The function $f:\{1,2,3\}\to\{x,y,z,p\}$ such that $f=\{(1,x),(2,y),(3,z)\}$ is a one-one function.

SECTION B
*(Question numbers 21 to 25 carry **2 marks** each.)*

21. Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$.

OR

Find the domain of $\sin^{-1}(x^2-4)$.

22. Find the interval/s in which the function $f:R\to R$ defined by $f(x)=xe^x$, is increasing.

23. If $f(x)=\frac{1}{4x^2+2x+1}$; $x\in R$, then find the maximum value of $f(x)$.

OR

Find the maximum profit that a company can make, if the profit function is given by $P(x)=72+42x-x^2$, where x is the number of units and P is the profit in rupees.

24. Evaluate : $\int_{-1}^{1}\log_e\left(\frac{2-x}{2+x}\right)dx$.

25. Check whether the function $f:R\to R$ defined by $f(x)=x^3+x$, has any critical point/s or not? If yes, then find the point/s.

SECTION C

*(Question numbers 26 to 31 carry **3 marks** each.)*

26. Evaluate : $\int \frac{2x^2+3}{x^2(x^2+9)}dx;\ x \neq 0$.

27. The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some real number:

$$P(X)=\begin{cases} k, \text{ if } x=0 \\ 2k, \text{ if } x=1 \\ 3k, \text{ if } x=2 \\ 0, \text{ otherwise} \end{cases}.$$

(i) Determine the value of k.

(ii) Find $P(X<2)$.

(iii) Find $P(X>2)$.

28. Evaluate : $\int \sqrt{\frac{x}{1-x^3}}\,dx;\ x \in (0, 1)$.

OR

Evaluate : $\int_0^{\frac{\pi}{4}} \log_e(1+\tan x)\,dx$.

29. Solve the differential equation : $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}}+y^2\right)dy,\ (y \neq 0)$.

OR

Solve the differential equation : $(\cos^2 x)\frac{dy}{dx}+y=\tan x;\ \left(0 \le x \le \frac{\pi}{2}\right)$.

30. Solve the following Linear Programming graphically.

Minimize $z=x+2y$.

Subject to the constraints $x+2y \ge 100,\ 2x-y \le 0,\ 2x+y \le 200,\ x,\ y \ge 0$.

OR

Solve the following Linear Programming graphically.

Maximize $z=-x+2y$.

Subject to the constraints $x \ge 3,\ x+y \ge 5,\ x+2y \ge 6,\ y \ge 0$.

31. If $(a+bx)e^{\frac{y}{x}}=x$, then prove that $x\frac{d^2y}{dx^2}=\left(\frac{a}{a+bx}\right)^2$.

SECTION D

*(Question numbers 32 to 35 carry **5 marks** each.)*

32. Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2+1,\ 0 \le y \le x+1,\ 0 \le x \le 2\}$ and find the area of the region, using the method of integration.

33. Let N be the set of all natural numbers and R be a relation on $N \times N$ defined by $(a, b)R(c, d) \Leftrightarrow ad=bc$ for all $(a, b), (c, d) \in N \times N$.

Show that R is an equivalence relation on $N \times N$.

Also, find the equivalence class of $(2, 6)$, i.e., $[(2, 6)]$.

OR

Show that the function $f: R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \dfrac{x}{1+|x|}$, $x \in R$ is one-one and onto function.

34. Using the matrix method, solve the following system of linear equations :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.$$

35. Find the coordinates of the image of the point $(1, 6, 3)$ with respect to the line $\vec{r}=(\hat{j}+2\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$; where '$\lambda$' is a scalar.
Also, find the distance of the image from the y-axis.

OR

An aeroplane is flying along the line $\vec{r}=\lambda(\hat{i}-\hat{j}+\hat{k})$; where '$\lambda$' is a scalar and another aeroplane is flying along the line $\vec{r}=\hat{i}-\hat{j}+\mu(-2\hat{j}+\hat{k})$; where '$\mu$' is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them.

SECTION E

(Question numbers 36 to 38 carry ***4 marks*** *each.)*
This section contains ***three Case-study / Passage based questions****.*
First two questions have ***three sub-parts*** *(i), (ii) and (iii) of* ***marks 1, 1 and 2*** *respectively.*
Third question has ***two sub-parts*** *of* ***2 marks*** *each.*

36. **CASE STUDY I :** Read the following passage and then answer the questions given below.
In an office three employees James, Sophia and Oliver process incoming copies of a certain form. James processes 50% of the forms, Sophia processes 20% and Oliver the remaining 30% of the forms. James has an error rate of 0.06, Sophia has an error rate of 0.04 and Oliver has an error rate of 0.03.

(i) Find the probability that Sophia processed the form and committed an error.
(ii) Find the total probability of committing an error in processing the form.
(iii) The manager of the Company wants to do a quality check. During inspection, he selects a form at random from the days output of processed form. If the form selected at random has an error, find the probability that the form is not processed by James.

OR

(iii) Let E be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that James, Sophia and Oliver processed the form. Find the value of $\sum_{i=1}^{3} P(E_i|E)$.

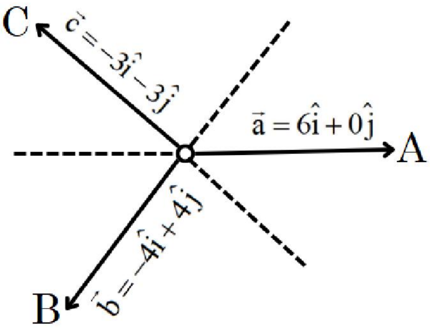
37. **CASE STUDY II :** Read the following passage and then answer the questions given below.

Teams A, B and C went for playing a tug of war game. Teams A, B and C have attached a rope to a metal ring and are trying to pull the ring into their own area.

Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}$ kN.

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}$ kN.

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}$ kN.



(i) What is the magnitude of the force of Team A?

(ii) Which team will win the game?

(iii) Find the magnitude of the resultant force exerted by the teams.

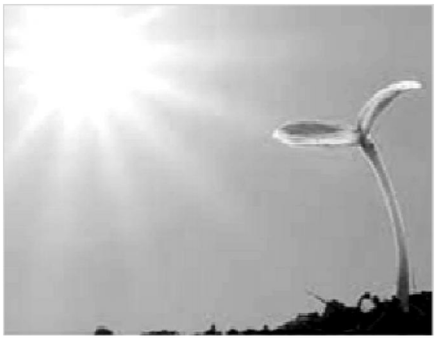
OR

(iii) In what direction is the ring getting pulled?

38. **CASE STUDY III :** Read the following passage and then answer the questions given below.

The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation

$y = 4x - \frac{1}{2}x^2$, where 'x' is the number of days exposed to the sunlight, for $x \le 3$.



(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(ii) Does the rate of growth of the plant increase or decrease in the first three days? What will be the height of the plant after 2 days?

📖 Detailed Solutions for CBSE Sample Paper (2023-24)

SECTION A

01. (d) Note that $A=\begin{bmatrix}0&1\\1&0\end{bmatrix}$ $\quad \therefore A^2=A.A=\begin{bmatrix}1&0\\0&1\end{bmatrix}$.

02. (d) The statement "$(A+B)^{-1}=B^{-1}+A^{-1}$" is not correct.

03. (b) Area = Magnitude of $\frac{1}{2}\begin{vmatrix}-3&0&1\\3&0&1\\0&k&1\end{vmatrix}$

$$\Rightarrow \pm 9=\frac{1}{2}\begin{vmatrix}-3&0&1\\3&0&1\\0&k&1\end{vmatrix}$$

Expanding along C_2, we get $\pm 18=-0+0-k(-3-3)$

$\Rightarrow k=\pm 3$.

04. (a) Since f is continuous at $x=0$.

Therefore, $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x)=f(0)$

$\Rightarrow \lim\limits_{x\to 0^-}\frac{-kx}{x}=\lim\limits_{x\to 0^+}3=3$

$\Rightarrow \lim\limits_{x\to 0^-}(-k)=3$

$\Rightarrow (-k)=3 \quad \therefore k=-3$.

05. (d) Note that $6\hat{i}+9\hat{j}-18\hat{k}=3(2\hat{i}+3\hat{j}-6\hat{k})$.

That means, $2\hat{i}+3\hat{j}-6\hat{k}$ and $6\hat{i}+9\hat{j}-18\hat{k}$ are parallel.

Also the fixed point $\hat{i}+\hat{j}-\hat{k}$ on the line $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$ does not satisfy $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(6\hat{i}+9\hat{j}-18\hat{k})$; where λ and μ are scalars.

06. (c) For the D.E. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}\right)^2$, the higher order derivative is $\frac{d^2y}{dx^2}$.

Clearly the degree is 2.

07. (b) $Z=px+qy$

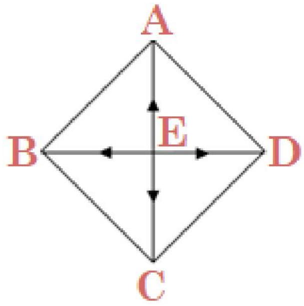
At $(3, 0)$, $Z=3p \quad \ldots$(i)

At $(1, 1)$, $Z=p+q \quad \ldots$(ii)

From (i) and (ii), $3p=p+q$

$\Rightarrow 2p=q \quad \therefore p=\frac{q}{2}$.

08. (a) $ABCD$ is a rhombus whose diagonals bisect each other. Consider the diagram.



That is, $|\overrightarrow{EA}|=|\overrightarrow{EC}|$ and $|\overrightarrow{EB}|=|\overrightarrow{ED}|$.

But since they are opposite to each other so, they are of opposite signs.

That is, $\overrightarrow{EA}=-\overrightarrow{EC}$ and $\overrightarrow{EB}=-\overrightarrow{ED}$.

$\Rightarrow \overrightarrow{EA}+\overrightarrow{EC}=\vec{0}$ ...(i)

and $\overrightarrow{EB}+\overrightarrow{ED}=\vec{0}$ ...(ii)

Adding (i) and (ii), we get $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}=\vec{0}$.

09. (b) Let $f(x)=e^{\sin^2 x}\cos^3(2n+1)x$

$\because f(\pi-x)=e^{\sin^2(\pi-x)}\cos^3(2n+1)(\pi-x)=-e^{\sin^2 x}\cos^3(2n+1)x=-f(x)$

$\therefore \int_0^\pi e^{\sin^2 x}\cos^3(2n+1)x\,dx=0.$

Recall that, if f is integrable in $[0, 2a]$ and $f(2a-x)=-f(x)$, then $\int_0^{2a} f(x)\,dx=0.$

10. (b) Matrix A is a skew symmetric matrix of odd order (order of A is 3) $\therefore |A|=0.$

11. (c) We observe that $(0, 0)$ does not satisfy the inequality $x-y\ge 1$.
So, the half plane represented by $x-y\ge 1$ will not contain origin therefore, it will not contain the shaded feasible region.

12. (b) Vector component of $\vec{a}$ along $\vec{b}=(\vec{a}.\hat{b})\hat{b}=\left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}=\dfrac{18}{25}(3\hat{j}+4\hat{k}).$

13. (d) $|\text{adj.}(2A)|=|(2A)|^2=(2^3|A|)^2=2^6|A|^2=2^6\times(-2)^2=2^8.$

14. (d) Let A, B, C be the respective events of solving the problem by three students.
Then $P(A)=\dfrac{1}{2}$, $P(B)=\dfrac{1}{3}$ and $P(C)=\dfrac{1}{4}$ $\quad\therefore P(\overline{A})=\dfrac{1}{2}$, $P(\overline{B})=\dfrac{2}{3}$ and $P(\overline{C})=\dfrac{3}{4}$.
Here A, B, C are independent events.
$\because$ Problem is solved if at least one of them solves the problem.
$\therefore$ Required probability is $=P(A\cup B\cup C)=1-P(\overline{A})P(\overline{B})P(\overline{C})$

$$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=1-\frac{1}{4}=\frac{3}{4}.$$

Alternatively,
The problem will be solved if one or more of them can solve the problem.
Therefore, required probability is

$$P(A\overline{B}\overline{C})+P(\overline{A}B\overline{C})+P(\overline{A}\overline{B}C)+P(AB\overline{C})+P(A\overline{B}C)+P(\overline{A}BC)+P(ABC)$$

$$=\frac{1}{2}.\frac{2}{3}.\frac{3}{4}+\frac{1}{2}.\frac{1}{3}.\frac{3}{4}+\frac{1}{2}.\frac{2}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{3}{4}+\frac{1}{2}.\frac{2}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{1}{4}=\frac{3}{4}.$$

15. (c) $ydx-xdy=0$

$\Rightarrow ydx=xdy$

$\Rightarrow \dfrac{dy}{y}=\dfrac{dx}{x}$

On integrating, we get $\int\dfrac{dy}{y}=\int\dfrac{dx}{x}$

$\Rightarrow \log_e|y|=\log_e|x|+\log_e|c|$

Since $x, y, c>0$, we write $\log_e y=\log_e x+\log_e c$

$\Rightarrow \log_e y=\log_e(cx)$

$\Rightarrow y=cx.$

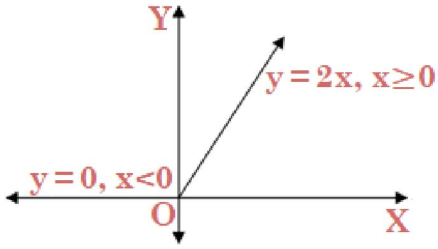
16. (d) Since Dot product of two perpendicular vectors is zero.

$\therefore (2\hat{i}-\hat{j}+2\hat{k}).(3\hat{i}+\lambda\hat{j}+\hat{k})=0$

$\Rightarrow 2\times3+(-1)\lambda+2\times1=0$

$\Rightarrow \lambda=8.$

17. (c) $f(x)=x+|x|=\begin{cases}2x, & x\ge 0\\ 0, & x<0\end{cases}$



Let $y=f(x)$

Consider the diagram (graph) shown.

There is a sharp corner at $x=0$, so $f(x)$ is not differentiable at $x=0$.

Alternatively,

$f(x)=x+|x|=\begin{cases}2x, & x\ge 0\\ 0, & x<0\end{cases} \quad \Rightarrow f'(x)=\begin{cases}2, & x\ge 0\\ 0, & x<0\end{cases}$

$\because Lf'(0)=0$ and $Rf'(0)=2$

$\therefore$ Function f is not differentiable at $x=0$.

For $x\ge 0$, $f(x)=2x$ (linear function); when $x<0$, $f(x)=0$ (constant function).

Hence, $f(x)$ is differentiable only when $x\in(-\infty, 0)\cup(0, \infty)$.

18. (d) We know that, $l^2+m^2+n^2=1$

$\Rightarrow\left(\frac{1}{c}\right)^2+\left(\frac{1}{c}\right)^2+\left(\frac{1}{c}\right)^2=1$

$\Rightarrow 3\left(\frac{1}{c}\right)^2=1 \quad \Rightarrow c^2=3$

$\Rightarrow c=\pm\sqrt{3}$.

19. (a) Given $\frac{d}{dx}\big(f(x)\big)=(x-1)^3(x-3)^2$

Note that $\frac{d}{dx}(f(x))<0,\ \forall x\in(1-h, 1)$

and $\frac{d}{dx}\big(f(x)\big)>0,\ \forall x\in(1, 1+h)$

Clearly, **A** and **R** both are true, also **R** is correct explanation of **A**.

20. (d) **A** is false. Since the element 4 has no image under f. So the relation f is not a function. That means, f can't be a bijective function.

Moreover, **R** is true. The given function f is one-one, because for each element $\in\{1, 2, 3\}$, there is a different image in $\{x, y, z, p\}$ under f.

SECTION B

21. $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)=\sin^{-1}\cos\left(6\pi+\frac{3\pi}{5}\right)=\sin^{-1}\cos\left(\frac{3\pi}{5}\right)=\frac{\pi}{2}-\cos^{-1}\cos\left(\frac{3\pi}{5}\right)$

$=\frac{\pi}{2}-\frac{3\pi}{5}=-\frac{\pi}{10}$.

OR

For $\sin^{-1}(x^2-4)$ to be defined, we must have $-1\le(x^2-4)\le 1$

$\Rightarrow 3\le x^2\le 5$

$\Rightarrow \sqrt{3}\le|x|\le\sqrt{5}$

$\Rightarrow x\in\left[-\sqrt{5}, -\sqrt{3}\right]\cup\left[\sqrt{3}, \sqrt{5}\right]$.

So, domain is $\left[-\sqrt{5}, -\sqrt{3}\right]\cup\left[\sqrt{3}, \sqrt{5}\right]$.

22. $f(x) = x e^x$

$\Rightarrow f'(x) = e^x (x+1)$

For $f'(x) = e^x (x+1) = 0$, we get $x = -1$

When $x \in [-1, \infty)$, $(x+1) \geq 0$ and $e^x > 0$

$\Rightarrow f'(x) \geq 0$

$\therefore f(x)$ increases in $x \in [-1, \infty)$.

23. Given $f(x) = \dfrac{1}{4x^2 + 2x + 1}$

Let $g(x) = \dfrac{1}{f(x)} = 4x^2 + 2x + 1$

$\Rightarrow g(x) = 4\left(x^2 + 2x\dfrac{1}{4} + \dfrac{1}{16}\right) + \dfrac{3}{4}$

$\Rightarrow g(x) = 4\left(x + \dfrac{1}{4}\right)^2 + \dfrac{3}{4} \geq \dfrac{3}{4}$

$\therefore$ Maximum value of $f(x) = \dfrac{4}{3}$.

Alternatively,

Given $f(x) = \dfrac{1}{4x^2 + 2x + 1}$

Let $g(x) = \dfrac{1}{f(x)} = 4x^2 + 2x + 1$

$\Rightarrow g'(x) = 8x + 2$ and $g''(x) = 8$

For $g'(x) = 0,\ 8x + 2 = 0 \quad \Rightarrow x = -\dfrac{1}{4}$

$\because g''\left(x = -\dfrac{1}{4}\right) = 8 > 0$

$\therefore g(x)$ is minimum when $x = -\dfrac{1}{4}$.

So, $f(x)$ is maximum at $x = -\dfrac{1}{4}$.

$\therefore$ Maximum value of $f(x) = f\left(-\dfrac{1}{4}\right) = \dfrac{1}{4\left(-\dfrac{1}{4}\right)^2 + 2\left(-\dfrac{1}{4}\right) + 1} = \dfrac{4}{3}$.

Note that, if you do not take $g(x) = \dfrac{1}{f(x)}$ and directly proceed with differentiation of $f(x)$ then too this problem can be solved.

OR

Given $P(x) = 72 + 42x - x^2$, where profit P is in the rupees (₹).

$\therefore P'(x) = 42 - 2x$ and $P''(x) = -2$

For maxima and minima, $P'(x) = 0$, $42 - 2x = 0 \qquad \Rightarrow x = 21$

$\because P''(21) = -2 < 0$

So, $P(x)$ is maximum at $x = 21$.

The maximum value of $P(x)=P(21)=72+(42\times 21)-(21)^2=513$.

Therefore, the maximum profit is ₹513.

24. Let $f(x)=\log_e\left(\dfrac{2-x}{2+x}\right)$

Note that $f(-x)=\log_e\left(\dfrac{2+x}{2-x}\right)=-\log_e\left(\dfrac{2-x}{2+x}\right)=-f(x)$

That means, $f(x)$ is an odd function.

$\therefore \int_{-1}^{1}\log_e\left(\dfrac{2-x}{2+x}\right)dx=0$.

Recall that, if f is integrable in $[-a, a]$ and $f(-x)=-f(x)$, then $\int_{-a}^{a}f(x)\,dx=0$.

25. $f(x)=x^3+x$, for all $x\in R$.

$\therefore f'(x)=3x^2+1$

Since for all $x\in R$, $x^2\geq 0$

$\therefore f'(x)>0$

Hence, no critical point exists for $f(x)$.

SECTION C

26. Take $x^2=t$.

Then $\dfrac{2x^2+3}{x^2(x^2+9)}=\dfrac{2t+3}{t(t+9)}=\dfrac{A}{t}+\dfrac{B}{t+9}$

$\Rightarrow 2t+3=A(t+9)+Bt$

On comparing both sides, we get $9A=3,\ A+B=2$

On solving, we get $A=\dfrac{1}{3}$ and $B=\dfrac{5}{3}$.

Now $\int\dfrac{2x^2+3}{x^2(x^2+9)}dx=\dfrac{1}{3}\int\dfrac{dx}{x^2}+\dfrac{5}{3}\int\dfrac{dx}{x^2+9}=-\dfrac{1}{3x}+\dfrac{5}{9}\tan^{-1}\left(\dfrac{x}{3}\right)+c$.

27. (i) Recall that $\sum P(X=r)=1$

$\Rightarrow P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+...=1$

$\Rightarrow k+2k+3k+0+0+...=1$

$\Rightarrow k=\dfrac{1}{6}$.

(ii) $P(X<2)=P(X=0)+P(X=1)=k+2k=3k=3\times\dfrac{1}{6}=\dfrac{1}{2}$.

(iii) $P(X>2)=P(X=3)+P(X=4)+...$

$\therefore P(X>2)=0$.

28. Let $x^{\frac{3}{2}}=t\Rightarrow\sqrt{x}\,dx=\dfrac{2}{3}dt$.

Now $\int\sqrt{\dfrac{x}{1-x^3}}dx=\int\dfrac{\sqrt{x}}{\sqrt{1-(x^{3/2})^2}}dx=\dfrac{2}{3}\int\dfrac{dt}{\sqrt{1-t^2}}$

$=\dfrac{2}{3}\sin^{-1}(t)+c=\dfrac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right)+c$.

OR

Let $I=\int_0^{\frac{\pi}{4}}\log_e(1+\tan x)\,dx$ ...(i)

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log_e\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$ (Using $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log_e\left(1+\frac{1-\tan x}{1+\tan x}\right)dx=\int_0^{\frac{\pi}{4}}\log_e\left(\frac{2}{1+\tan x}\right)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log_e 2dx-\int_0^{\frac{\pi}{4}}\log_e(1+\tan x)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\log_e 2dx-I$ (Using (i)

$\Rightarrow 2I=\log_e 2[x]_0^{\pi/4}$

$\Rightarrow 2I=\log_e 2\left[\frac{\pi}{4}-0\right]$

$\Rightarrow I=\frac{\pi}{8}\log_e 2.$

29. $ye^{\frac{x}{y}}dx=\left(xe^{\frac{x}{y}}+y^2\right)dy$

$\Rightarrow \frac{dx}{dy}=\frac{xe^{\frac{x}{y}}+y^2}{ye^{\frac{x}{y}}}$

$\Rightarrow \frac{dx}{dy}=\frac{x}{y}+\frac{y}{e^{\frac{x}{y}}}$ ...(i)

Put $x=vy\Rightarrow\frac{dx}{dy}=v+y\frac{dv}{dy}$.

So equation (i) becomes $v+y\frac{dv}{dy}=v+\frac{y}{e^v}$

$\Rightarrow y\frac{dv}{dy}=\frac{y}{e^v}$ $\Rightarrow e^v dv=dy$

On integrating, we get $\int e^v dv=\int dy$

$\Rightarrow e^v=y+c$

$\Rightarrow e^{x/y}=y+c$.

OR

$(\cos^2 x)\frac{dy}{dx}+y=\tan x$

Dividing both the sides by $\cos^2 x$, we get $\frac{dy}{dx}+\frac{y}{\cos^2 x}=\frac{\tan x}{\cos^2 x}$

$\frac{dy}{dx}+y(\sec^2 x)=\tan x\sec^2 x \quad \ldots(i)$

Comparing with $\frac{dy}{dx}+P(x)y=Q(x)$, we get $P(x)=\sec^2 x,\ Q(x)=\tan x\sec^2 x$

The integrating factor will be, $\text{I.F.}=e^{\int \sec^2 x\,dx}=e^{\tan x}$

Required solution is given as $y(e^{\tan x})=\int (e^{\tan x})\tan x\sec^2 x dx+c$

Put $\tan x=t \Rightarrow \sec^2 x dx=dt$ in the integral in RHS.

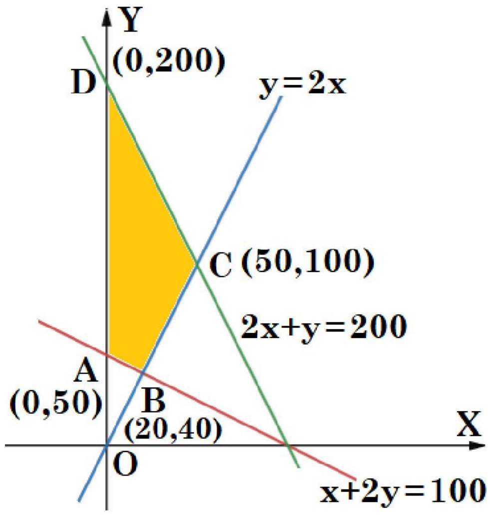
$\therefore y e^{\tan x}=\int t e^t dt+c$

$\Rightarrow y e^{\tan x}=t e^t-e^t+c$

$\Rightarrow y e^{\tan x}=(\tan x-1)e^{\tan x}+c$

$\therefore y=(\tan x-1)+c e^{-\tan x}$.

30. Graph with the feasible region for the given constraints is given below.

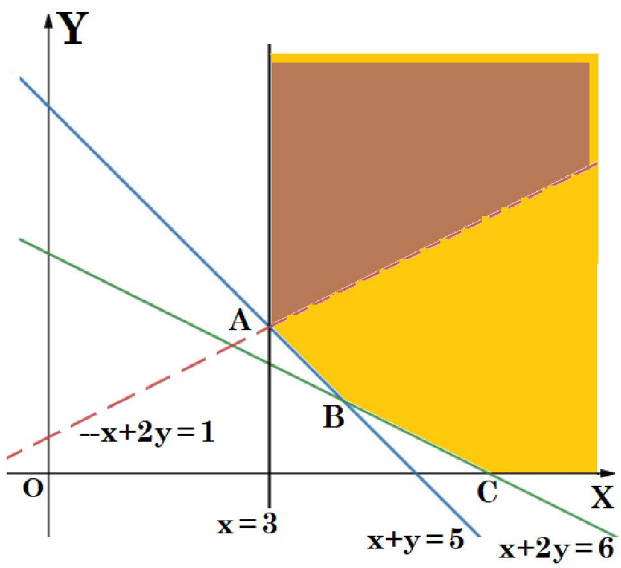


Corner point	Value of Z	
A(0, 50)	100	Minimum
B(20, 40)	100	Minimum
C(50, 100)	250	
D(0, 200)	400	

The minimum value of Z is 100, at all the points on the line segment joining the points (0, 50) and (20, 40).

OR

Consider the graph shown with feasible region for the given constraints.
Note that the corner points are $A(3, 2)$, $B(4, 1)$ and $C(6, 0)$.



Corner point	Value of Z	
A(3, 2)	1	Maximum
B(4, 1)	–2	
C(6, 0)	–6	

Observe that the feasible region obtained is unbounded.
That means, $Z=1$ may or may not be the maximum value.
To check, let $-x+2y>1$.
It is clearly evident that the resulting open half-plane $-x+2y>1$ has points in common with the feasible region.
Hence, $Z=1$ is **not** the maximum value.
We conclude, Z has **no** maximum value.

31. $(a+bx)e^{y/x}=x \qquad \Rightarrow e^{y/x}=\dfrac{x}{a+bx}$

On taking logarithm both the sides, we get $\dfrac{y}{x}=\log_e\left(\dfrac{x}{a+bx}\right)=\log_e x-\log_e(a+bx)$

On differentiating with respect to x, $\dfrac{x\dfrac{dy}{dx}-y}{x^2}=\dfrac{1}{x}-\dfrac{1}{a+bx}\times\dfrac{d}{dx}(a+bx)$

$\Rightarrow \dfrac{x\dfrac{dy}{dx}-y}{x^2}=\dfrac{1}{x}-\dfrac{b}{a+bx}$

$\Rightarrow x\dfrac{dy}{dx}-y=x^2\left(\dfrac{1}{x}-\dfrac{b}{a+bx}\right)=\dfrac{ax}{a+bx}$

On differentiating again with respect to x, $x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}-\dfrac{dy}{dx}=\dfrac{(a+bx)a-ax(b)}{(a+bx)^2}$

$\Rightarrow x\dfrac{d^2y}{dx^2}=\left(\dfrac{a}{a+bx}\right)^2.$

SECTION D

32. Consider $y=x^2+1$, $y=x+1$.

We need to find the point of intersection of the curve $y=x^2+1$ and the line $y=x+1$.

We write $x^2+1=x+1$

$\Rightarrow x(x-1)=0$

$\Rightarrow x=0,1.$

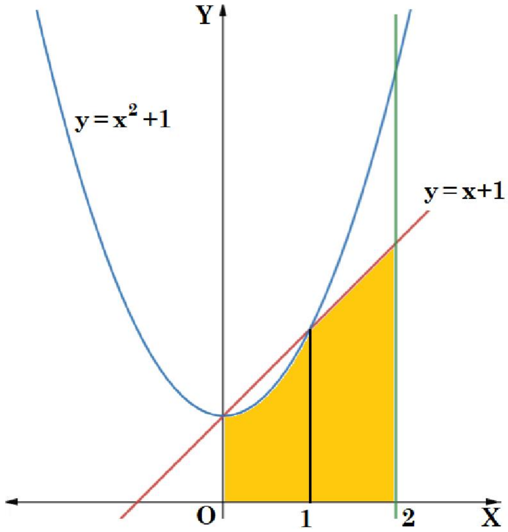
So, the point of intersections are (0, 1) and (1, 2).

Required area $=\int_0^1 (x^2+1)\,dx+\int_1^2 (x+1)\,dx$

$=\left[\dfrac{x^3}{3}+x\right]_0^1+\left[\dfrac{x^2}{2}+x\right]_1^2$

$=\left[\left(\dfrac{1}{3}+1\right)-0\right]+\left[(2+2)-\left(\dfrac{1}{2}+1\right)\right]$

$=\dfrac{23}{6}$ Sq. units.



33. Let (a, b) be an arbitrary element of $N\times N$. Then, $(a, b)\in N\times N$ and $a, b\in N$.

We have, $ab=ba$.

$\because a, b\in N$ and the multiplication is commutative on N.

$\Rightarrow (a, b)\,R\,(a, b)$, according to the definition of the relation R on $N\times N$.

Thus $(a, b)\,R\,(a, b)\ \forall (a, b)\in N\times N$.

So, R is reflexive relation on $N\times N$.

Let $(a, b), (c, d)$ be arbitrary elements of $N\times N$ such that $(a, b)\,R\,(c, d)$.

Then, $(a, b)\,R\,(c, d) \quad \Rightarrow ad=bc$

$\Rightarrow bc=ad$

$\Rightarrow cb=da$

$\Rightarrow (c, d)\,R\,(a, b)$

Thus, $(a, b) R (c, d) \Rightarrow (c, d) R (a, b)$

So, R is symmetric relation on $N \times N$.

Let $(a, b), (c, d), (e, f)$ be arbitrary elements of $N \times N$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$.

Thus, $(a, b) R (c, d) \Rightarrow ad = bc$ and $(c, d) R (e, f) \Rightarrow cf = de$

Consider $(ad)(cf) = (bc)(de)$

$\Rightarrow af = be$

$\Rightarrow (a, b) R (e, f)$

Thus $(a, b) R (c, d)$ and $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$

So, R is transitive relation on $N \times N$.

As the relation R is reflexive, symmetric and transitive so, it is equivalence relation on $N \times N$.

$[(2, 6)] = \{(x, y) \in N \times N : (x, y) R (2, 6)\}$

$= \{(x, y) \in N \times N : 6x = 2y\}$

$= \{(x, y) \in N \times N : 3x = y\}$

$= \{(x, 3x) : x \in N\}$

$= \{(1, 3), (2, 6), (3, 9), ...\}$.

OR

Here $f : R \to A$, where $A = \{x \in R : -1 < x < 1\}$, is defined by $f(x) = \dfrac{x}{1+|x|}$, $x \in R$.

One-one : Let $x_1, x_2 \in R$.

Also let $f(x_1) = f(x_2)$.

That is, $\dfrac{x_1}{1+|x_1|} = \dfrac{x_2}{1+|x_2|}$

Case I : If $x_1, x_2 > 0$ then, $\dfrac{x_1}{1+x_1} = \dfrac{x_2}{1+x_2}$

$\Rightarrow x_1 + x_1x_2 = x_2 + x_1x_2$

$\Rightarrow x_1 = x_2 \ldots$(i)

Case II : If $x_1, x_2 < 0$ then, $\dfrac{x_1}{1-x_1} = \dfrac{x_2}{1-x_2}$

$\Rightarrow x_1 - x_1x_2 = x_2 - x_1x_2$

$\Rightarrow x_1 = x_2 \ldots$(ii)

Case III : If $x_1 > 0, x_2 < 0$ then, clearly $x_1 \neq x_2$. Therefore, $\dfrac{x_1}{1+x_1} \neq \dfrac{x_2}{1-x_2}$

$\Rightarrow f(x_1) \neq f(x_2) \ldots$(iii)

Case IV : If $x_1 < 0, x_2 > 0$ then, clearly $x_1 \neq x_2$. Therefore, $\dfrac{x_1}{1-x_1} \neq \dfrac{x_2}{1+x_2}$

$\Rightarrow f(x_1) \neq f(x_2) \ldots$(iv)

By (i), (ii), (iii) and (iv), it is evident that the function f is one-one.

Onto : Let $y \in A \ \therefore -1 < y < 1$ so that $y = f(x)$.

Recall that, $A = \{x \in R : -1 < x < 1\}$.

Now $y = \dfrac{x}{1+|x|}$ $\qquad \Rightarrow y = \dfrac{x}{1 \pm x}$

$\Rightarrow y \pm xy = x$

$\Rightarrow y = x \mp xy$

$\Rightarrow y = x(1 \mp y)$

$\Rightarrow x = \dfrac{y}{1 \mp y} \in R$ for all $-1 < y < 1$.

That is, for all f-image in the Codomain A, we've a pre-image in the Domain R of the function f. So, f is onto function.

34. The given system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}.$$

Now $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 1200 \neq 0 \;\therefore A^{-1}$ exists.

$$\therefore \text{adj.} A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Hence, $A^{-1} = \dfrac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

Since $AX = B$

$\Rightarrow A^{-1}AX = A^{-1}B$

$\Rightarrow IX = A^{-1}B$

$\Rightarrow X = A^{-1}B$

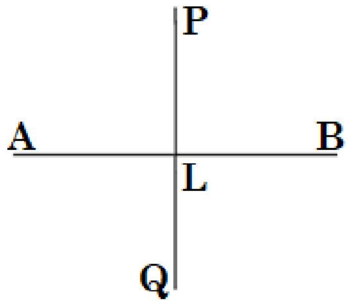
$$\Rightarrow X = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Thus, $\dfrac{1}{x} = \dfrac{1}{2}, \dfrac{1}{y} = \dfrac{1}{3}, \dfrac{1}{z} = \dfrac{1}{5}$

Hence, $x = 2,\ y = 3,\ z = 5$.

35. Let $P(1, 6, 3)$ be the given point, and let L be the foot of perpendicular from P to the given line AB (shown in the figure).



Let $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$.

The coordinates of a general point on the given line AB are given by $x=\lambda$, $y=2\lambda+1$ and $z=3\lambda+2$.

Let point L be given by $(\lambda,\ 2\lambda+1,\ 3\lambda+2)$.

So, direction ratios of PL are $\lambda-1,\ \ 2\lambda+1-6$ and $3\lambda+2-3$

That is, $\lambda-1,\ 2\lambda-5$ and $3\lambda-1$.

$\because$ Direction ratios of the given line are 1, 2, and 3; also line AB is perpendicular to PL.

$\therefore\ (\lambda-1)(1)+(2\lambda-5)(2)+(3\lambda-1)(3)=0$ (Using $a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow \lambda=1$

So, coordinates of L are (1, 3, 5).

Let $Q(x_1,\ y_1,\ z_1)$ be the image of $P(1,\ 6,\ 3)$ in the given line.

Then, L is the mid-point of PQ.

So, $\frac{x_1+1}{2}=1,\ \frac{y_1+6}{2}=3$ and $\frac{z_1+3}{2}=5$

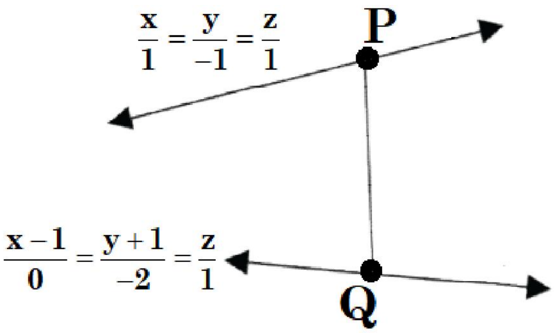
$\Rightarrow x_1=1,\ y_1=0$ and $z_1=7$

Hence, the image of $P(1,\ 6,\ 3)$ in the given line AB is $Q(1,\ 0,\ 7)$.

Now, the distance of the point $Q(1,\ 0,\ 7)$ from the y-axis is $\sqrt{1^2+7^2}=\sqrt{50}$ units.

OR

Consider the following diagram.



The equation of two given straight lines in the Cartesian form are $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1}$ ...(i) and $\frac{x-1}{0}=\frac{y+1}{-2}=\frac{z}{1}$ ...(ii)

Note that the lines are not parallel as their direction ratios are not proportional.

Let P be a point on the line (i) and Q be a point on the line (ii) such that line PQ is perpendicular to both of the lines.

Let $P(\lambda,-\lambda,\ \lambda)$ be any random point on the line (i).

Also let $Q(1,-2\mu-1,\ \mu)$ be the random point on line (ii).

Then the direction ratios of the line PQ are $\lambda-1,\ -\lambda+2\mu+1,\ \lambda-\mu$.

Since PQ is perpendicular to the line (i), so we have $(\lambda-1).1+(-\lambda+2\mu+1).(-1)+(\lambda-\mu).1=0$

$\Rightarrow 3\lambda-3\mu=2$ ...(iii)

Since PQ is perpendicular to the line (ii), so we have $0.(\lambda-1)+(-\lambda+2\mu+1).(-2)+(\lambda-\mu).1=0$

$\Rightarrow 3\lambda-5\mu=2$ ...(iv)

Solving (iii) and (iv), we get $\mu=0,\ \lambda=\frac{2}{3}$.

Therefore, the coordinates of point P are $\left(\frac{2}{3},-\frac{2}{3},\frac{2}{3}\right)$ and that of Q are $(1,-1,\ 0)$.

Hence, the required shortest distance PQ is given by $PQ=\sqrt{\left(1-\frac{2}{3}\right)^2+\left(-1+\frac{2}{3}\right)^2+\left(0-\frac{2}{3}\right)^2}$

$\Rightarrow PQ=\sqrt{\frac{2}{3}}$ units.

SECTION E

36. Let E_1, E_2 and E_3 denote the events that James, Sophia and Oliver processed the form, which are clearly pair wise mutually exclusive and exhaustive set of events.

Then $P(E_1)=\frac{50}{100}=\frac{5}{10}$, $P(E_2)=\frac{20}{100}=\frac{1}{5}$ and $P(E_3)=\frac{30}{100}=\frac{3}{10}$.

Also, let E be the event of committing an error.

We have, $P(E|E_1)=0.06$, $P(E|E_2)=0.04$ and $P(E|E_3)=0.03$.

(i) The probability that Sophia processed the form and committed an error is given by

$P(E\cap E_2)=P(E_2).P(E|E_2)=\frac{1}{5}\times 0.04$

$\therefore P(E\cap E_2)=0.008$.

(ii) The total probability of committing an error in processing the form is given by

$P(E)=P(E_1).P(E|E_1)+P(E_2).P(E|E_2)+P(E_3).P(E|E_3)$

$\Rightarrow P(E)=\frac{50}{100}\times 0.06+\frac{20}{100}\times 0.04+\frac{30}{100}\times 0.03$

$\therefore P(E)=0.047$.

(iii) The probability that the form is processed by James given that form has an error is given by

$$P(E_1|E)=\frac{P(E\,|\,E_1)\ P(E_1)}{P(E\,|\,E_1)\ P(E_1)+P(E\,|\,E_2)\ P(E_2)+P(E\,|\,E_3)\ P(E_3)}$$

$$\Rightarrow P(E_1\,|\,E)=\frac{0.06\times\frac{50}{100}}{0.06\times\frac{50}{100}+0.04\times\frac{20}{100}+0.03\times\frac{30}{100}}=\frac{30}{47}.$$

Therefore, the required probability that the form is not processed by James given that form has an error $=P(\overline{E}_1\,|\,E)=1-P(E_1|E)=1-\frac{30}{47}=\frac{17}{47}$.

OR

(iii) Recall that, the Sum of the posterior probabilities is 1.

So, $\sum_{i=1}^{3}P(E_i|E)=P(E_1|E)+P(E_2|E)+P(E_3|E)=1$

Let's show the proof of above statement.

Consider $P(E_1|E)+P(E_2|E)+P(E_3|E)=\frac{P(E\cap E_1)}{P(E)}+\frac{P(E\cap E_2)}{P(E)}+\frac{P(E\cap E_3)}{P(E)}$

$\Rightarrow \quad =\frac{P(E\cap E_1)+P(E\cap E_2)+P(E\cap E_3)}{P(E)}$

$\Rightarrow \quad =\frac{P\left((E\cap E_1)\cup(E\cap E_2)\cup(E\cap E_3)\right)}{P(E)}$ [as E_i and E_j; $i\neq j$ are mutually exclusive events

$\Rightarrow \quad =\frac{P\left(E\cap(E_1\cup E_2\cup E_3)\right)}{P(E)}$

$\Rightarrow \quad = \dfrac{P(E\cap S)}{P(E)} = \dfrac{P(E)}{P(E)} = 1$; where S denote the sample space.

37. We have $\left|\vec{F_1}\right| = \sqrt{6^2+0^2} = 6$ kN,

$\left|\vec{F_2}\right| = \sqrt{(-4)^2+4^2} = \sqrt{32} = 4\sqrt{2}$ kN,

$\left|\vec{F_3}\right| = \sqrt{(-3)^2+(-3)^2} = \sqrt{18} = 3\sqrt{2}$ kN.

(i) Magnitude of force of Team $A = 6$ kN.

(ii) Since, 6 kN is largest so, team A will win the game.

(iii) As $\vec{F} = \vec{F_1}+\vec{F_2}+\vec{F_3} = 6\hat{i}+0\hat{j}-4\hat{i}+4\hat{j}-3\hat{i}-3\hat{j} = -\hat{i}+\hat{j}$

$\therefore \left|\vec{F}\right| = \sqrt{(-1)^2+(1)^2} = \sqrt{2}$ kN.

OR

(iii) As $\vec{F} = \vec{F_1}+\vec{F_2}+\vec{F_3} = -\hat{i}+\hat{j}$

To find the direction in which the ring is getting pulled, we shall find the angle of resultant force $\vec{F}$ with the x-axis.

Note that the direction ratios of x-axis are 1, 0, 0.

Also for $\vec{F}$, the direction ratios are $-1, 1, 0$.

$$\therefore \cos\theta = \frac{1(-1)+0(1)+0(0)}{\sqrt{1^2+0^2+0^2}\sqrt{(-1)^2+1^2+0^2}} = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

$$\therefore \theta = \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

[where θ is the angle made by the resultant force with the positive direction of the x-axis

38. Given that $y = 4x - \frac{1}{2}x^2$

(i) Rate of growth of the plant with respect to the number of days exposed to sunlight is given by $\dfrac{dy}{dx} = 4 - x$.

(ii) Let rate of growth be represented by the function $g(x) = \dfrac{dy}{dx}$.

Now, $g'(x) = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) \quad \Rightarrow g'(x) = \dfrac{d}{dx}(4-x) = -1$

$\therefore g'(x) = -1 < 0$

$\Rightarrow g(x)$ decreases.

So the rate of growth of the plant decreases for the first three days.

Height of the plant after 2 days is given by $y = 4\times 2 - \frac{1}{2}(2)^2 = 6$ cm.

PLEASURE TEST SERIES

For CBSE 2024 Board Exams - Class 12

MATHEMATICS (041) PTS-01

a compilation by
O.P. GUPTA
INDIRA AWARD WINNER

Time Allowed : 180 Minutes	**Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (4 **marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

SECTION A

*(Question numbers 01 to 20 carry **1 mark** each.)*
*Followings are **multiple choice questions**. Select the correct option in each one of them.*

01. If $A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ and $(3I+4A)(3I-4A)=x^2 I$, then the value (s) of x is/are

(a) ± 25 (b) 0 (c) ± 5 (d) 25

02. If $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, then $A^{2024}=$

(a) $\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ (b) $\begin{bmatrix}0 & 2024\\ 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ (d) $\begin{bmatrix}2024 & 0\\ 0 & 2024\end{bmatrix}$

03. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

04. If the vector $\hat{i}-b\hat{j}+\hat{k}$ is equally inclined to the coordinate axes, then the value of b is

(a) -1 (b) 1 (c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

05. If $\frac{d}{dx}\left(f(x)\right)=\log x$, then $f(x)$ equals

(a) $x(\log x-x)+c$ (b) $x(\log x-1)+c$ (c) $x(\log x+x)+c$ (d) $\frac{1}{x}+c$

06. The general solution of the differential equation $x\,dy-(1+x^2)\,dx=dx$ is

(a) $y = 2x + \frac{x^3}{3} + c$ (b) $y = 2\log x + \frac{x^3}{3} + c$

(c) $y = \frac{x^2}{2} + c$ (d) $y = 2\log x + \frac{x^2}{2} + c$

07. The number of corner points of the feasible region determined by the constraints $x - y \ge 0$, $2y \le x + 2$, $x \ge 0$, $y \ge 0$ is

(a) 2 (b) 3 (c) 4 (d) 5

08. In ΔABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. Let D is the mid-point of BC, then vector $\overrightarrow{AD}$ is equal to

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$ (c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

09. $\int_0^{\frac{\pi}{6}} \sec^2\left(x - \frac{\pi}{6}\right) dx$ is equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$

10. If $|A| = |kA|$, where A is a non-singular square matrix of order 2×2, then sum of all possible values of k is

(a) 1 (b) -1 (c) 2 (d) 0

11. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to

(a) 40 (b) 96 (c) 144 (d) 136

12. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to

(a) $2\Delta^2$ (b) $4\Delta^2$ (c) 2Δ (d) 4Δ

13. If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $(x + y + z)$ is

(a) 10 (b) 6 (c) 8 (d) 0

14. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is

(a) $\frac{1}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{18}$ (d) $\frac{1}{2}$

15. What is the product of the order and degree of the differential equation

$\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

(a) 3 (b) 2 (c) 6 (d) not defined

16. The function $f(x) = x|x|$ is

(a) continuous and differentiable at $x = 0$

(b) continuous but not differentiable at $x = 0$
(c) differentiable but not continuous at $x = 0$
(d) neither differentiable nor continuous at $x = 0$

17. The value of λ for which the angle between the lines $\vec{r} = \hat{i} + \hat{j} + \hat{k} + p(2\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{r} = (1+q)\hat{i} + (1+q\lambda)\hat{j} + (1+q)\hat{k}$ is $\frac{\pi}{2}$, is

(a) -4 (b) 4 (c) 2 (d) -2

18. If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0

Followings are ***Assertion-Reason based questions****.*
In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).
Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A) :** The range of the function $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

20. **Assertion (A) :** A line through the points $(4,7,8)$ and $(2,3,4)$ is parallel to a line through the points $(-1,-2,1)$ and $(1,2,5)$.

Reason (R) : Lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are parallel, if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

SECTION B

(Question numbers 21 to 25 carry ***2 marks*** *each.)*

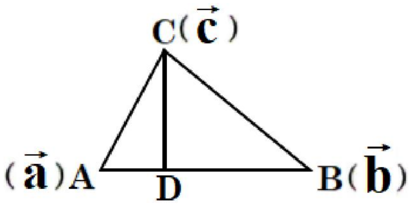
21. Draw the graph of $f(x) = \sin^{-1}x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of $f(x)$.

OR

A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

22. Show that the function $f(x) = \frac{16\sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

23. Let A, B and C are non-collinear points with position vectors $\vec{a}$, $\vec{b}$ and, $\vec{c}$ respectively.



Show that the length of perpendicular (CD) drawn from C on AB is $\frac{|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}{|\vec{b} - \vec{a}|}$.

OR

If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z + \frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$, then find the relation between α and β.

24. If $f(x)=\begin{cases} ax+b; & 0<x\le 1 \\ 2x^2-x; & 1<x<2 \end{cases}$ is a differentiable function in $(0, 2)$, then find the values of a and b.

25. If $\vec{r}=3\hat{i}-2\hat{j}+6\hat{k}$, then find the value of $(\vec{r}\times\hat{j})\cdot(\vec{r}\times\hat{k})-12$.

SECTION C

*(Question numbers 26 to 31 carry **3 marks** each.)*

26. Evaluate $\int_0^{\frac{\pi}{2}}[\log(\sin x)-\log(2\cos x)]dx$.

27. Chandrayaan, India's lunar exploration program designed by ISRO, has two types of missions : those focused on orbiter missions and those focused on lander and rover missions.

Historically, 70% of Chandrayaan missions have been orbiters, and 30% have been lander and rover missions.
Due to various technical challenges, orbiter missions have a success rate of 80%, while lander and rover missions have a success rate of 60%.

If a Chandrayaan mission is randomly selected and it is known to be successful, then what is the probability that it was an orbiter mission?

OR

Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

28. Find $\int\frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}dx$.

OR

Find $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx$.

29. Find the general solution of the differential equation : $\frac{d}{dx}(xy^2)=2y(1+x^2)$.

OR

Solve the differential equation : $\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y\,dx=\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x\,dy$.

30. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}}\frac{1}{(e^x+e^{-x})(e^x-e^{-x})}dx$.

31. Solve the following linear programming problem graphically.

Maximize $z=5x+3y$

Subject to the constraints $3x+5y\le 15$, $5x+2y\le 10$, $x\ge 0$, $y\ge 0$.

SECTION D

*(Question numbers 32 to 35 carry **5 marks** each.)*

32. Determine the area of the region bounded by the curves $x^2=y$, $y=x+2$ and x-axis, using the concept of integration.

OR

It is given that the area of the region bounded by the line $y = mx\ (m > 0)$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

33. A relation R is defined on a set of real number $\mathbb{R}$ as

$$R = \{(x, y) : x.y \text{ is an irrational number}\}.$$

Check whether R is reflexive, symmetric and transitive or not.

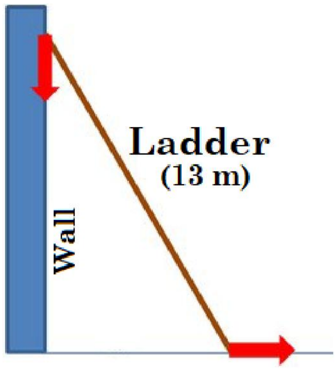
OR

A function $f : [-4, 4] \rightarrow [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.

34. A ladder of 13 m length, is leaning against a wall.

The foot of the ladder is pulled along the ground away from the wall, at the rate of 1.5 m/s.

How fast is the angle between the ladder and the ground changing, when the foot of the ladder is 12 m away from the wall? Use derivatives.



35. If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find the product AB.

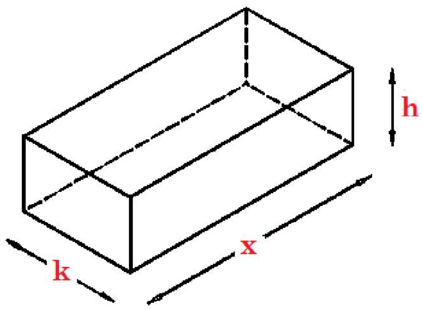
Hence, use the product AB to solve the following system of equations.

$$x - 2y = 3,\ 2x - y - z = 2,\ -2y + z = 3.$$

SECTION E

*(Question numbers 36 to 38 carry **4 marks** each.)*
*This section contains **three Case-study / Passage based questions**.*
*First two questions have **three sub-parts** (i), (ii) and (iii) of **marks 1, 1 and 2** respectively.*
*Third question has **two sub-parts** of **2 marks** each.*

36. **CASE STUDY I :** Read the following passage and then answer the questions given below.



A foreign client approaches ISHA BRICKS COMPANY for a special type of bricks.
The client requests for few samples of bricks as per their requirement.
The solid rectangular brick is to be made from 1 cubic feet of clay of special type.
The brick must be 3 times as long as it is wide.
(i) According to the figure shown, the length of brick is 'x', width is 'k' and height is 'h'.
Obtain an expression in terms of 'h' and 'k'.

(ii) Express the surface area (S) of the brick, as a function of 'k'.

(iii) Find $\frac{dS}{dk}$. At what value of k, $\frac{dS}{dk} = 0$?

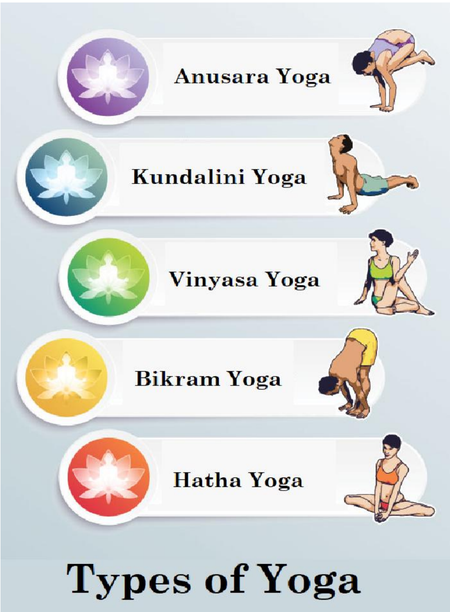
Show that $\frac{d^2S}{dk^2}$ is positive, at this obtained value of k. What does it signify?

OR

(iii) Find the minimum value of S, using second derivative test.

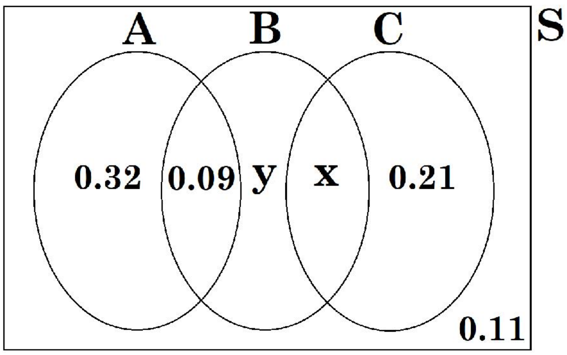
37. **CASE STUDY II :** Read the following passage and then answer the questions given below.

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:



The venn diagram below represents the probabilities of three different types of Yoga A, B and C performed by the people of a society.

Further, it is given that probability of a member performing type C Yoga is 0.44.



(i) Find the value of x.

(ii) Find the value of y.

(iii) Find $P(A|B)$ and $P(C|B)$.

OR

(iii) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

38. **CASE STUDY III :** Read the following passage and then answer the questions given below.

The Indian Cost Guard (ICG) while patrolling, saw a suspicious boat with some men. They were not looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. One of the officer observed that the boat is moving along a plane surface.

At an instant, the coordinates of the position of coast guard helicopter and boat are at the points A(2, 3, 5) and B(1, 4, 2) respectively.

(i) Write the direction cosines of line AB.

(ii) When the position of coast guard helicopter is at the point C(1, 0, –3), then the position of the boat is at the point D(3, –2, 3). Check if the line CD is parallel to line AB. Justify.

📖 Detailed Solutions (PTS-09)

SECTION A

01. (d) Let $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$\Rightarrow A=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore A^2=AA=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow A^2=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

02. (a) As $A=\begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix so, $|A|=\begin{vmatrix} k & 8 \\ 4 & 2k \end{vmatrix}=0$

$\Rightarrow 2k^2-32=0$

$\Rightarrow k^2=16$

$\therefore k=\pm 4$.

As $k>0$ so, $k=4$.

03. (a) $\because (x-1)\hat{i}+12\hat{j}-3\hat{k}=5\hat{i}+2x\hat{j}-3\hat{k}$

By comparing the coefficients of $\hat{i},\ \hat{j},\ \hat{k}$, we get $x-1=5,\ 12=2x$

$\therefore x=6$.

04. (b) As f is continuous at $x=0$ so, $\lim\limits_{x\to 0} f(x)=f(0)$ i.e., $\lim\limits_{x\to 0}\dfrac{1-\cos(ax)}{x\ \sin x}=\dfrac{1}{2}$

$$\Rightarrow \lim_{x\to 0}\frac{2\sin^2\frac{ax}{2}}{x^2\left(\frac{\sin x}{x}\right)}=\frac{1}{2}$$

$$\Rightarrow 2\lim_{(ax/2)\to 0}\left(\frac{\sin^2\frac{ax}{2}}{\frac{a^2x^2}{4}}\right)\times\frac{a^2}{4}\times\lim_{x\to 0}\frac{1}{\left(\frac{\sin x}{x}\right)}=\frac{1}{2}$$

$$\Rightarrow 2(1)^2\times\frac{a^2}{4}\times\frac{1}{1}=\frac{1}{2}$$

$\Rightarrow a^2=1$

$\Rightarrow a=\pm 1$

But $a<0$ so, $a=-1$.

05. (d) $\displaystyle\int_0^a \frac{1}{1+4x^2}dx=\frac{\pi}{8}$

$$\Rightarrow \int_0^a \frac{1}{1+(2x)^2}dx=\frac{\pi}{8}$$

$$\Rightarrow \frac{1}{2}\left[\tan^{-1}2x\right]_0^a=\frac{\pi}{8}$$

$$\Rightarrow \tan^{-1}2a-\tan^{-1}0=\frac{\pi}{4}$$

$\Rightarrow 2a = \tan\frac{\pi}{4} = 1$

$\therefore a = \frac{1}{2}$.

06. (a) $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3} = \frac{y^{1/3}}{x^{1/3}}$

$\Rightarrow y^{-1/3}dy = x^{-1/3}dx$

$\Rightarrow \int y^{-1/3}dy = \int x^{-1/3}dx$

$\Rightarrow \frac{3}{2}y^{2/3} = \frac{3}{2}x^{2/3} + k$

$\therefore y^{2/3} - x^{2/3} = C$, where $C = \frac{2k}{3}$.

07. (c) Here $Z_{(4,10)} = 38,\ Z_{(6,8)} = 36,\ Z_{(0,8)} = 24,\ Z_{(6,5)} = 27$.

Clearly, minimum value of Z is '24' and it is obtained at (0, 8).

08. (a) Unit vector $= \frac{\hat{i} + 2\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + (2)^2 + (-2)^2}} = \frac{\hat{i} + 2\hat{j} - 2\hat{k}}{3}$.

09. (b) $\int_{\sqrt{3}}^{2\sqrt{2}} \frac{x}{\sqrt{x^2+1}}dx = \frac{1}{2}\int_{\sqrt{3}}^{2\sqrt{2}} \frac{2x}{\sqrt{x^2+1}}dx = \frac{1}{2}\times\left[2\sqrt{x^2+1}\right]_{\sqrt{3}}^{2\sqrt{2}}$

$= \left[\sqrt{x^2+1}\right]_{\sqrt{3}}^{2\sqrt{2}}$

$= \left[\sqrt{(2\sqrt{2})^2+1} - \sqrt{(\sqrt{3})^2+1}\right] = \sqrt{9} - \sqrt{4} = 3 - 2$

$= 1$.

10. (d) Consider $AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I$

$\Rightarrow \left(\frac{1}{6}A\right)B = I \qquad \therefore B^{-1} = \frac{1}{6}A$.

11. (d) Note that $Z_A = 90,\ Z_B = 60,\ Z_C = 180,\ Z_D = 180$.

As Z is maximum at C(15, 15) and D(0, 20) so, maximum value of Z is obtained at all the points of line segment CD.

12. (a) By def. of equality of matrices, we get : $2a + b = 4,\ a - 2b = -3,\ 5c - d = 11,\ 4c + 3d = 24$.

On solving the equations, we get : $a = 1,\ b = 2,\ c = 3,\ d = 4$.

Hence, $a + b - c + 2d = 1 + 2 - 3 + 2(4) = 8$.

13. (d) $\because |adj.A| = |A|^{n-1}$, where n is order of A.

$\therefore |adj.(2A)| = |2A|^{3-1} = |2A|^2 = \left[2^3|A|\right]^2 = \left[2^3 \times 4\right]^2 = 2^{10}$.

14. (b) Clearly, $P(A) = \frac{5}{26},\ P(B) = \frac{5}{13}$.

Also $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{5}$

$$\Rightarrow \frac{P(A)+P(B)-P(A\cup B)}{P(B)}=\frac{2}{5}$$

$$\Rightarrow \frac{5}{26}+\frac{5}{13}-P(A\cup B)=\frac{2}{5}\times\frac{5}{13}$$

$$\Rightarrow \frac{5}{26}+\frac{5}{13}-\frac{2}{1}\times\frac{1}{13}=P(A\cup B)$$

$$\therefore P(A\cup B)=\frac{11}{26}.$$

15. (d) $\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^4\right]=0$

$$\Rightarrow 4\left(\frac{d^2y}{dx^2}\right)^3\times\frac{d^3y}{dx^3}=0.$$

So, the degree is 1.

16. (c) On dividing both the sides by e^{x+y}, we get : $e^{-y}+e^{-x}=1$

So, $e^{-y}\left(-\frac{dy}{dx}\right)+e^{-x}(-1)=0$

$$\Rightarrow \left(-\frac{dy}{dx}\right)-e^{y-x}=0$$

$$\Rightarrow \frac{dy}{dx}=-e^{y-x}.$$

17. (b) Let $\vec{a}=\sqrt{2}\,\hat{i}+\hat{j}+\hat{k}$

$$\Rightarrow \hat{a}=\frac{\sqrt{2}\,\hat{i}+\hat{j}+\hat{k}}{\sqrt{2+1+1}}=\frac{\sqrt{2}\,\hat{i}+\hat{j}+\hat{k}}{2}$$

$\because \hat{a}=\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{2}+\frac{\hat{k}}{2}=\cos\alpha\hat{i}+\cos\beta\hat{j}+\cos\gamma\hat{k}$ (On comparing both sides)

$\therefore \cos\beta=\frac{1}{2}$, where β is the angle made by $\vec{a}$ with y-axis.

18. (a) As the given line is in the symmetric form so, it passes through (1, –1, 2) and its direction ratios are 1, 2, –1. So, the line in vector form is $\vec{r}=\hat{i}-\hat{j}+2\hat{k}+\lambda(\hat{i}+2\hat{j}-\hat{k})$.

19. (d) For the lines, $\vec{b}_1=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}_2=\hat{i}+\hat{k}$.

So, the angle between the lines is $\cos\theta=\frac{\left|(\hat{i}+\hat{j}-\hat{k}).(\hat{i}+\hat{k})\right|}{\sqrt{1+1+1}\sqrt{1+1}}$

$$\Rightarrow \cos\theta=\frac{0}{\sqrt{6}}=0$$

$$\therefore \theta=\frac{\pi}{2}.$$

So, A is false.
Also, note that R is true.

20. (c) As $(1,2)\in S$ so, if $(2,1)$ is added to the relation S then, it becomes symmetric relation.
That is, A is true.
Also, R is false.

SECTION B

21. Let $y=\tan\left(\dfrac{1}{2}\cos^{-1}\dfrac{2}{\sqrt5}\right)$ $\quad\left[\text{Put } \cos^{-1}\dfrac{2}{\sqrt5}=\theta\Rightarrow\cos\theta=\dfrac{2}{\sqrt5}\right.$

$$\Rightarrow y=\tan\left(\frac{\theta}{2}\right)=\frac{\sin(\theta/2)}{\cos(\theta/2)}$$

$$\Rightarrow y=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\frac{1-\frac{2}{\sqrt5}}{1+\frac{2}{\sqrt5}}}=\sqrt{\frac{\sqrt5-2}{\sqrt5+2}}$$

$$\Rightarrow y=\sqrt{\frac{\sqrt5-2}{\sqrt5+2}.\frac{\sqrt5-2}{\sqrt5-2}}=\sqrt{\frac{(\sqrt5-2)^2}{5-4}}$$

$\therefore y=\sqrt5-2$.

OR

$f(x)=\cos x\ \forall x\in\mathbb{R}$

As $f\left(\dfrac{\pi}{4}\right)=\cos\left(\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt2}$ and, $f\left(-\dfrac{\pi}{4}\right)=\cos\left(-\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt2}$

$\Rightarrow f\left(\dfrac{\pi}{4}\right)=f\left(-\dfrac{\pi}{4}\right)$ but, $\dfrac{\pi}{4}\neq-\dfrac{\pi}{4}$

$\therefore f(x)$ is not one-one.

Also we know that the range of cos x is [–1, 1] i.e., $f(x)=\cos x\in[-1,1]\ \forall x\in\mathbb{R}$.

Note that codomain $\mathbb{R}$ of $f(x)=\cos x$ is not same as the range of $f(x)$, which is [–1, 1].

So, $f(x)$ is not onto.

22. $f(x)=\sqrt3\sin x-\cos x-2mx+n$

$\Rightarrow f'(x)=\sqrt3\cos x+\sin x-2m$

$\Rightarrow f'(x)=2\left(\dfrac{\sqrt3}{2}\cos x+\sin x\times\dfrac{1}{2}\right)-2m$

$\Rightarrow f'(x)=2\left(\sin\dfrac{\pi}{3}\cos x+\sin x\cos\dfrac{\pi}{3}\right)-2m$

$\Rightarrow f'(x)=2\sin\left(\dfrac{\pi}{3}+x\right)-2m$

As f (x) is decreasing on R so, $f'(x)\le0$ i.e., $2\sin\left(\dfrac{\pi}{3}+x\right)-2m\le0$

$\therefore m\ge\sin\left(\dfrac{\pi}{3}+x\right)$ …(i)

We know that for all $x\in R$, $\sin\left(\dfrac{\pi}{3}+x\right)\in[-1,1]$ i.e., $-1\le\sin\left(\dfrac{\pi}{3}+x\right)\le1$ …(ii)

By (i) and (ii), we conclude that $m\ge1$.

23. Let $\vec a=2\hat i-4\hat j-5\hat k$ and $\vec b=2\hat i+2\hat j+3\hat k$

So, the diagonal of the parallelogram are

$\vec d_1=\vec a+\vec b=4\hat i-2\hat j-2\hat k$ and $\vec d_2=\vec a-\vec b$ or, $\vec b-\vec a=-6\hat j-8\hat k$ or, $6\hat j+8\hat k$

Therefore the unit vectors parallel to the diagonals are

$\hat{d}_1 = \dfrac{4\hat{i}-2\hat{j}-2\hat{k}}{2\sqrt{6}} = \dfrac{2\hat{i}-\hat{j}-\hat{k}}{\sqrt{6}}$ and $\hat{d}_2 = \dfrac{-6\hat{j}-8\hat{k}}{10} = \dfrac{-3\hat{j}-4\hat{k}}{5}$ or, $\dfrac{6\hat{j}+8\hat{k}}{10} = \dfrac{3\hat{j}+4\hat{k}}{5}$.

OR

Writing the Cartesian equation of the line, $\dfrac{x-1}{3-1} = \dfrac{y-2}{3-2} = \dfrac{z+3}{2-(-3)}$

That is, $\dfrac{x-1}{2} = \dfrac{y-2}{1} = \dfrac{z+3}{5} = \lambda$ *say*.

So, parametric equations of the line are $x = 2\lambda+1,\ y = \lambda+2,\ z = 5\lambda-3$.

24. $y = (\sin x)^x$

$\Rightarrow y = e^{\log_e (\sin x)^x}$

$\Rightarrow y = e^{x \log_e (\sin x)}$

$\therefore \dfrac{dy}{dx} = e^{x\log_e (\sin x)} \times \left[x \times \dfrac{1}{\sin x} \times \cos x + \log_e (\sin x) \times 1 \right]$

$\therefore \dfrac{dy}{dx} = (\sin x)^x \left[x\cot x + \log_e (\sin x) \right]$.

25. As $\left|\vec{a}+\vec{b}\right|^2 + \left|\vec{a}-\vec{b}\right|^2 = (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) + (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$

$$= \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}.\vec{b} + \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\vec{a}.\vec{b} = 2\left[\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 \right]$$

So, $\left|\vec{a}+\vec{b}\right|^2 + \left|\vec{a}-\vec{b}\right|^2 = 2\left[\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 \right]$

$\Rightarrow 60^2 + 40^2 = 2\left[22^2 + \left|\vec{b}\right|^2 \right]$

$\Rightarrow \left|\vec{b}\right|^2 = 2116$.

Hence, $\left|\vec{b}\right| = 46$.

SECTION C

26. $\displaystyle\int \frac{x-3}{(x-1)^3} e^x\, dx = \int \left\{ \frac{x-1}{(x-1)^3} - \frac{2}{(x-1)^3} \right\} e^x\, dx$

$$= \int \left\{ \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right\} e^x\, dx$$

$$= \frac{e^x}{(x-1)^2} + C.$$

\# Note that $\int \{f(x) + f'(x)\}\, e^x\, dx = f(x)e^x + C$, here $f(x) = \dfrac{1}{(x-1)^2}$ and $f'(x) = -\dfrac{2}{(x-1)^3}$.

27. Let E_1 : selection of a person of blood group O,

E_2 : selection of a person of other blood group, and

E : selection of a left handed person.

$\therefore P(E_1) = \dfrac{30}{100},\ P(E_2) = \dfrac{70}{100},\ P(E|E_1) = \dfrac{6}{100},\ P(E|E_2) = \dfrac{10}{100}$.

By Bayes' theorem, $P(E_1|E) = \dfrac{P(E_1)\ P(E|E_1)}{P(E_1)\ P(E|E_1) + P(E_2)\ P(E|E_2)}$

$$\Rightarrow P(E_1 | E) = \frac{\frac{30}{100}\times\frac{6}{100}}{\frac{30}{100}\times\frac{6}{100}+\frac{70}{100}\times\frac{10}{100}} = \frac{18}{18+70}$$

$$\therefore P(E_1 | E) = \frac{18}{88} \text{ or, } \frac{9}{44}.$$

OR

(i) $P(1)+P(2)+P(3)+P(4)+P(5)+P(6)+P(7)=1$

$\Rightarrow 2k+3k+4k+5k+10k+12k+14k=1 \qquad \Rightarrow 50k=1$

$\therefore k=\frac{1}{50}.$

(ii) $E(X)=\sum_{i=1}^{n} x_i p_i = 1P(1)+2P(2)+3P(3)+4P(4)+5P(5)+6P(6)+7P(7)$

$$\Rightarrow E(X)=1\left(\frac{2}{50}\right)+2\left(\frac{3}{50}\right)+3\left(\frac{4}{50}\right)+4\left(\frac{5}{50}\right)+5\left(\frac{10}{50}\right)+6\left(\frac{12}{50}\right)+7\left(\frac{14}{50}\right)$$

$$\Rightarrow E(X)=\frac{20+20+50+72+98}{50}=\frac{260}{50}=5.2.$$

28. Let $I=\int_0^{\pi}\frac{x}{1+\sin x}dx$

$$\Rightarrow I=\int_0^{\pi}\frac{\pi-x}{1+\sin(\pi-x)}dx$$

$$\Rightarrow I=\int_0^{\pi}\frac{\pi-x}{1+\sin x}dx$$

$$\Rightarrow I=\int_0^{\pi}\frac{\pi}{1+\sin x}dx-\int_0^{\pi}\frac{x}{1+\sin x}dx$$

$$\Rightarrow I=\int_0^{\pi}\frac{\pi}{1+\sin x}dx-I$$

$$\Rightarrow 2I=\pi\int_0^{\pi}\frac{1}{1+\sin x}dx$$

$$\Rightarrow 2I=\pi\int_0^{\pi}\frac{1}{1+\sin x}dx$$

$$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$\Rightarrow I=\frac{\pi}{4}\int_0^{\pi}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx$$

$$\Rightarrow I=-\frac{\pi}{2}\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\pi}$$

$$\Rightarrow I=-\frac{\pi}{2}\left[\tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right)-\tan\left(\frac{\pi}{4}-0\right)\right]$$

$\Rightarrow I = -\frac{\pi}{2}[-1-1]$

$\therefore\ I = \pi$.

OR

$$\int_1^3 \left[|x| + |x-2|\right]dx = \int_1^3 |x|dx + \int_1^3 |x-2|dx$$

$$= \int_1^3 x\,dx + \int_1^2 |x-2|dx + \int_2^3 |x-2|dx$$

$$= \left[\frac{x^2}{2}\right]_1^3 + \int_1^2 (2-x)dx + \int_2^3 (x-2)dx$$

$$= \left[\frac{9}{2} - \frac{1}{2}\right] + \left[\frac{(2-x)^2}{2(-1)}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^3$$

$$= [4] - \frac{1}{2}[0-1] + \frac{1}{2}[1-0]$$

$$= 4 + \frac{1}{2} + \frac{1}{2}$$

$$= 5.$$

29. Rewriting the D.E., we have $\frac{dy}{dx} + \left(\frac{\cos x}{1+\sin x}\right)y = -\frac{x}{1+\sin x}$

On comparing with $\frac{dy}{dx} + P(x)y = Q(x)$, we have $P(x) = \frac{\cos x}{1+\sin x}$, $Q(x) = -\frac{x}{1+\sin x}$

Now I.F. $= e^{\int \frac{\cos x}{1+\sin x}dx} = e^{\log(1+\sin x)} = 1+\sin x$.

So, the solution is given as $y(1+\sin x) = \int \frac{-x}{1+\sin x} \times (1+\sin x)dx + C$

$\Rightarrow y(1+\sin x) = -\int x dx + C = -\frac{x^2}{2} + C$

Therefore, the general solution is $y(1+\sin x) = -\frac{x^2}{2} + C$.

$\because y(0) = 1 \qquad \therefore 1(1+\sin 0) = -\frac{0^2}{2} + C$

$\Rightarrow C = 1$

Hence required particular solution is $y(1+\sin x) = -\frac{x^2}{2} + 1$ or, $y = \frac{1}{1+\sin x} - \frac{x^2}{2+2\sin x}$.

OR

$x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right)$, $x \neq 0$

$\Rightarrow \frac{dy}{dx} = \frac{xy + 1 + \cos\left(\frac{y}{x}\right)}{x^2} \ldots\text{(i)}$

Putting $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i), we get : $v + x\frac{dv}{dx} = \frac{vx^2 + 1 + \cos\left(\frac{vx}{x}\right)}{x^2}$

$$\Rightarrow v + x\frac{dv}{dx} = v + \frac{1+\cos v}{x^2}$$

$$\Rightarrow \int \frac{dv}{1+\cos v} = \int \frac{dx}{x^3}$$

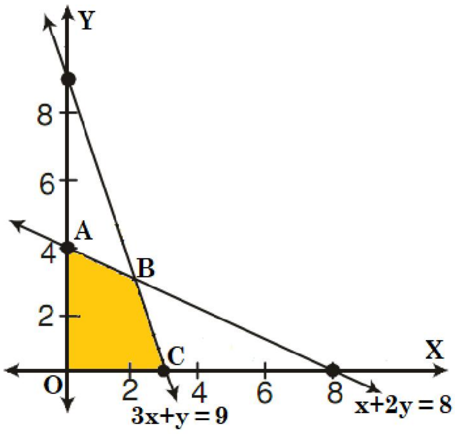
$$\Rightarrow \frac{1}{2}\int \sec^2\left(\frac{v}{2}\right)dv = \frac{x^{-3+1}}{-3+1} + C$$

$$\Rightarrow \frac{1}{2}\tan\left(\frac{v}{2}\right)\times 2 = -\frac{1}{2x^2} + C$$

$$\therefore \tan\left(\frac{y}{2x}\right) + \frac{1}{2x^2} = C.$$

30. Consider the graph shown here.

Corner Points	Value of Z
A(0, 4)	200
B(2, 3)	230 ← Maximum
C(3, 0)	120



So, the maximum value of Z is 230.
As $Z_{max} = 230$ is obtained at (2, 3).
Therefore, x coordinate is 2 and y coordinate is 3.

31. Let $I = \int \frac{1}{x(2+x^5)}dx$

Put $2 + x^5 = u \Rightarrow dx = \frac{du}{5x^4} \Rightarrow \frac{dx}{x} = \frac{du}{5x^5}$

$$\therefore I = \int \frac{du}{5x^5 u} = \int \frac{du}{5(u-2)u}$$

Consider $\frac{1}{5(u-2)u} = \frac{A}{u-2} + \frac{B}{u}$

$$\Rightarrow \frac{1}{5} = Au + B(u-2)$$

On comparing the coefficients of like terms, we get : $A = \frac{1}{10}, B = -\frac{1}{10}$

So, $I = \int\left(\frac{1}{10}\times\frac{1}{u-2} - \frac{1}{10}\times\frac{1}{u}\right)du$

$$\Rightarrow I = \frac{1}{10}\times\log|u-2| - \frac{1}{10}\times\log|u| + C$$

$$\Rightarrow I = \frac{1}{10}\times\left\{\log|u-2| - \log|u|\right\} + C$$

$$\Rightarrow I = \frac{1}{10}\times\log\left|\frac{u-2}{u}\right| + C$$

$$\therefore I = \frac{1}{10}\times\log\left|\frac{x^5}{2+x^5}\right| + C.$$

SECTION D

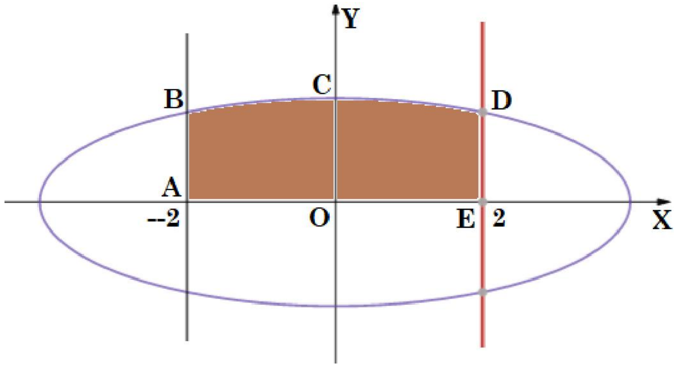
32. Given ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$.

Also, the equations of latus-rectums are $x=-2,\ x=2$.

Also, $\frac{y^2}{12}=1-\frac{x^2}{16}=\frac{16-x^2}{16}$

$\Rightarrow y^2=\frac{12}{16}(16-x^2)$

$\Rightarrow y=\pm\frac{\sqrt{3}}{2}\sqrt{16-x^2}$



Required area $=$ ar(ABCDEOA)

$\Rightarrow \quad =\int_{-2}^{2}\frac{\sqrt{3}}{2}\sqrt{16-x^2}\,dx$

$\Rightarrow \quad =\frac{\sqrt{3}}{2}\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_{-2}^{2}$

$\Rightarrow \quad =\frac{\sqrt{3}}{2}\left[\left(\sqrt{12}+8\sin^{-1}\frac{1}{2}\right)-\left(-\sqrt{12}+8\sin^{-1}\left(-\frac{1}{2}\right)\right)\right]$

$\Rightarrow \quad =\frac{\sqrt{3}}{2}\left[\left(\sqrt{12}+8\times\frac{\pi}{6}\right)-\left(-\sqrt{12}-8\times\frac{\pi}{6}\right)\right]$

$\Rightarrow \quad =\left(6+\frac{4\pi}{\sqrt{3}}\right)$ Sq. units .

33. We have $R=\{(x,y): x,y\in A,\ x \text{ and } y \text{ are either both odd or both even}\}$ and, $A=\{1,2,3,4,5,6,7,8,9\}$.

Reflexivity : Let any element $a\in A$. Clearly 'a' must be either odd or even, so that $(a,\ a)\in R$.
So, R is reflexive.

Symmetry : Let $(a,\ b)\in R$. That means, both 'a' and 'b' must be either odd or even.
That implies, $(b,\ a)\in R$.
So, R is symmetric.

Transitivity : Let $(a,\ b)\in R$ and $(b,\ c)\in R$.
Then, all elements a, b, c, must be either even or odd simultaneously.
That implies, $(a,\ c)\in R$.
Hence, R is a transitive relation.

Since the relation R is reflexive, symmetric and transitive so, it is an equivalence relation.
Now let $(1,\ x)\in R$, clearly x will be odd.
Hence, $[1]=\{1,\ 3,\ 5,\ 7,\ 9\}$.
Similarly, $[3]=[5]=[7]=[9]=\{1,\ 3,\ 5,\ 7,\ 9\}$.
Also let $(2,\ y)\in R$, clearly y will be even.
Hence, $[2]=\{2,\ 4,\ 6,\ 8\}$.
Similarly, $[2]=[4]=[6]=[8]=\{2,\ 4,\ 6,\ 8\}$.

OR

The relation R is defined as $(a,b)\in R \Leftrightarrow 1+ab>0\ \forall a,b\in\mathbb{R}$.

Reflexive : Let $a\in\mathbb{R}$. As $a^2\geq 0$ i.e., $1+a^2>0$ i.e., $1+a.a>0$ i.e., $(a,a)\in R$ so, R is reflexive.

Symmetric : Let $a,b\in\mathbb{R}$ and let $(a,b)\in R$. So, $1+ab>0$ i.e., $1+ba>0$ i.e., $(b,a)\in R$.
$\therefore$ R is symmetric.

Transitive : Let $a, b, c \in \mathbb{R}$. Let $(a,b) \in R$ and $(b,c) \in R$.

Put $a=1,\ b=-\frac{1}{2},\ c=-1$.

Note that $\left(1,-\frac{1}{2}\right) \in R$ as $1+1\left(-\frac{1}{2}\right)=\frac{1}{2}>0$.

Similarly, $\left(-\frac{1}{2},-1\right) \in R$ as $1+\left(-\frac{1}{2}\right)(-1)=\frac{3}{2}>0$

But $(1,-1) \notin R$ as $1+(1)(-1)=0>0$.

Hence, R is not transitive.

34. For $\vec{r}=(-2\hat{i}+3\hat{j})+\lambda(4\hat{i}-6\hat{j}+12\hat{k})$; $\vec{a}_1=-2\hat{i}+3\hat{j},\ \vec{b}_1=4\hat{i}-6\hat{j}+12\hat{k}$

For $\vec{r}=(2\hat{i}+3\hat{j}+2\hat{k})+\mu(2\hat{i}-3\hat{j}+6\hat{k})$; $\vec{a}_2=2\hat{i}+3\hat{j}+2\hat{k},\ \vec{b}_2=2\hat{i}-3\hat{j}+6\hat{k}$

Note that, $\vec{b}_1=2\vec{b}_2$ so, clearly both the lines are parallel.

Let $\vec{b}_2=2\hat{i}-3\hat{j}+6\hat{k}=\vec{b}$.

Now $\vec{a}_2-\vec{a}_1=(2\hat{i}+3\hat{j}+2\hat{k})-(-2\hat{i}+3\hat{j})=4\hat{i}+2\hat{k}$ and $(\vec{a}_2-\vec{a}_1)\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 0 & 2\\ 2 & -3 & 6\end{vmatrix}=6\hat{i}-20\hat{j}-12\hat{k}$.

$$\therefore \text{S.D.}=\frac{\left|(\vec{a}_2-\vec{a}_1)\times\vec{b}\right|}{\left|\vec{b}\right|}$$

$$\Rightarrow \quad =\frac{\left|6\hat{i}-20\hat{j}-12\hat{k}\right|}{\left|2\hat{i}-3\hat{j}+6\hat{k}\right|}=\frac{\sqrt{36+400+144}}{\sqrt{4+9+36}}=\frac{\sqrt{580}}{7}\text{ units}.$$

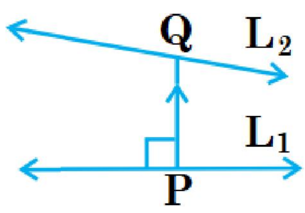
OR

Since Isha wants to travel from a point P on one path to a point Q on another path so, she must walk along the 'line of shortest distance'.

Let $L_1: \frac{x-6}{1}=\frac{2-y}{2}=\frac{z-2}{2}=\lambda$ (say) and $L_2: \frac{x+4}{3}=\frac{y}{-2}=\frac{z+1}{-2}=\mu$ (say).

Here vectors parallel to lines L_1 and L_2 are respectively,

$\vec{b}_1=\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}_2=3\hat{i}-2\hat{j}-2\hat{k}$.



Clearly the line of S.D. meets the given lines L_1 and L_2 at P and Q respectively.

So, the coordinates of any random point on the lines are given as :

$P(6+\lambda, 2-2\lambda, 2+2\lambda)$ and $Q(-4+3\mu, -2\mu, -1-2\mu)$.

The d.r.'s of PQ are $\lambda-3\mu+10,\ -2\lambda+2\mu+2,\ 2\lambda+2\mu+3$.

That is, a vector parallel to line PQ, $\vec{b}=(\lambda-3\mu+10)\hat{i}+(-2\lambda+2\mu+2)\hat{j}+(2\lambda+2\mu+3)\hat{k}$.

Since the line of S.D. (line PQ) is perpendicular to both the given lines.

So by using $\vec{b}.\vec{b}_1=0$ and, $\vec{b}.\vec{b}_2=0$, we get : $3\lambda-\mu+4=0$ and $3\lambda-17\mu+20=0$

On solving these eqs. we get : $\lambda=-1,\ \mu=1$.

$\therefore$ Coordinates of the points of intersection of line of shortest distance and given lines are $P(5,4,0)$ and $Q(-1,-2,-3)$.

Therefore, the equation of S.D. (line PQ) is : $\frac{x-5}{-1-5}=\frac{y-4}{-2-4}=\frac{z-0}{-3-0}$

That is, $\dfrac{x-5}{2}=\dfrac{y-4}{2}=\dfrac{z-0}{1}$.

35. For the matrix $A=\begin{bmatrix}1&3&4\\2&1&2\\5&1&1\end{bmatrix}$, we have $|A|=\begin{vmatrix}1&3&4\\2&1&2\\5&1&1\end{vmatrix}=-1+24-12=11\neq 0\ \therefore A^{-1}$ exists.

Consider A_{ij} be the cofactor of element a_{ij}.

$a_{11}=-1,\ a_{12}=8,\ a_{13}=-3,\ a_{21}=1,\ a_{22}=-19,\ a_{23}=14,\ a_{31}=2,\ a_{32}=6,\ a_{33}=-5$

$$\therefore \text{adj.}A=\begin{bmatrix}-1&1&2\\8&-19&6\\-3&14&-5\end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{|A|}\times \text{adj.}A=\frac{1}{11}\begin{bmatrix}-1&1&2\\8&-19&6\\-3&14&-5\end{bmatrix}$$

Now $x+3y+4z=8$, $2x+y+2z=5$ and $5x+y+z=7$

Let $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $B=\begin{bmatrix}8\\5\\7\end{bmatrix}$

Since $AX=B$ $\quad\therefore X=A^{-1}B=\dfrac{1}{11}\begin{bmatrix}-1&1&2\\8&-19&6\\-3&14&-5\end{bmatrix}\begin{bmatrix}8\\5\\7\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

By equality of matrices, we get : $x=1,\ y=1,\ z=1$.

SECTION E

36. **(i)** Note that $CD=r$, $VC=h$ and $VD=x$.

Also semi-vertical angle of the cone is, $\angle CVD=\dfrac{\pi}{6}$.

In ΔVCD, $\dfrac{CD}{VD}=\sin\dfrac{\pi}{6}$

$\Rightarrow \dfrac{r}{x}=\dfrac{1}{2}$

$\Rightarrow 2r=x$.

(ii) In ΔVCD, $\dfrac{CD}{VC}=\tan\dfrac{\pi}{6}$

$\Rightarrow \dfrac{r}{h}=\dfrac{1}{\sqrt{3}}$

$\Rightarrow \sqrt{3}\,r=h$.

(iii) As the volume of cone, $V=\dfrac{1}{3}\pi r^2 h$

$$\Rightarrow V = \frac{1}{3}\pi\left(\frac{x^2}{4}\right)\left(\frac{\sqrt{3}x}{2}\right) = \frac{\pi}{8\sqrt{3}}\times x^3 \qquad \left[\because r = \frac{x}{2},\ h = \sqrt{3}r = \frac{\sqrt{3}x}{2}\right.$$

$$\Rightarrow \frac{dV}{dt} = \frac{\pi}{8\sqrt{3}}\times 3x^2 \times \frac{dx}{dt}$$

$$\Rightarrow -1 = \frac{\pi}{8\sqrt{3}}\times 3x^2 \times \frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{8\sqrt{3}}{3\pi x^2}$$

$$\therefore \left.\frac{dx}{dt}\right]_{\text{at } x=4 \text{ cm}} = -\frac{8\sqrt{3}}{3\pi(4)^2} = -\frac{\sqrt{3}}{6\pi}\ \text{cm/s}.$$

Hence, the rate of decrease of slant height is $\frac{\sqrt{3}}{6\pi}$ cm/s.

OR

(iii) As the surface area of cone, $S = \pi r x$

$$\Rightarrow S = \frac{\pi}{2}x^2 \qquad \left[\because r = \frac{x}{2}\right.$$

$$\Rightarrow \frac{dS}{dt} = \pi \times x \times \frac{dx}{dt}$$

$$\Rightarrow -2 = \pi \times x \times \frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{2}{\pi x}$$

$$\therefore \left.\frac{dx}{dt}\right]_{\text{at } x=4\text{cm}} = -\frac{2}{4\pi}\ \text{cm/s} = -\frac{1}{2\pi}\ \text{cm/s}.$$

Hence, the rate of decrease of the slant height is $\frac{1}{2\pi}$ cm/s.

37. Let the events are defined as E_1 : Person chosen is a cyclist, E_2 : Person chosen is a scooter driver, E_3 : Person chosen is a car driver and, A : Person meets with an Accident.

Then, $P(E_1) = \frac{2000}{12000} = \frac{2}{12}$, $P(E_2) = \frac{4000}{12000} = \frac{4}{12}$, $P(E_3) = \frac{6000}{12000} = \frac{6}{12}$.

Also $P(A|E_1) = 0.01 = \frac{1}{100}$, $P(A|E_2) = 0.03 = \frac{3}{100}$, $P(A|E_3) = 0.15 = \frac{15}{100}$.

(i) Clearly, $P(E_2) = \frac{4000}{12000} = \frac{4}{12}$ i.e., $\frac{1}{3}$.

(ii) Clearly, $P(E_3) = \frac{6000}{12000} = \frac{6}{12}$ i.e., $\frac{1}{2}$.

(iii) $P(A) = P(A|E_1)\ P(E_1) + P(A|E_2)\ P(E_2) + P(A|E_3)\ P(E_3)$

$$\Rightarrow P(A) = \frac{1}{100}\times\frac{2}{12} + \frac{3}{100}\times\frac{4}{12} + \frac{15}{100}\times\frac{6}{12} = \frac{104}{1200}\ \text{or,}\ \frac{13}{150}.$$

OR

(iii) Using Bayes' theorem, $P(E_1|A)=\dfrac{P(A|E_1)\ P(E_1)}{P(A|E_1)\ P(E_1)+P(A|E_2)\ P(E_2)+P(A|E_3)\ P(E_3)}$

$$\Rightarrow P(E_1\,|\,A)=\dfrac{\dfrac{1}{100}\times\dfrac{1}{6}}{\dfrac{1}{100}\times\dfrac{1}{6}+\dfrac{3}{100}\times\dfrac{1}{3}+\dfrac{15}{100}\times\dfrac{1}{2}}=\dfrac{1}{52}.$$

38. **(i)** Since $C(x)=x^3-45x^2+600x$

$\Rightarrow \dfrac{d}{dx}[C(x)]=3x^2-90x+600$.

For $\dfrac{d}{dx}[C(x)]=C'(x)=0,\ 3x^2-90x+600=3(x-10)(x-20)=0$

$\Rightarrow (x-10)=0$ or, $(x-20)=0$

$\therefore x=10,\ 20$.

(ii) We have $C'(x)=3x^2-90x+600$ and $C''(x)=6x-90$.

For $C'(x)=3x^2-90x+600=0$

$\Rightarrow 3(x-10)(x-20)=0 \qquad \therefore x=10,\ 20$

Note that $C''(10)=-30<0$ and $C''(20)=30>0$.

So, $C(x)$ is minimum at $x=20$.

Therefore, the person must place the order for 20 trees in order to spend the least amount.

For CBSE 2024 Board Exams - Class 12

PLEASURE TEST SERIES

ANSWERS FOR PTS-14 TO PTS-20

a compilation by

O.P. GUPTA

INDIRA AWARD WINNER

Answers (PTS-14)

01.	(c)	**02.**	(b)	**03.**	(b)	**04.**	(d)	**05.**	(c)	**06.**	(b)	**07.**	(b)
08.	(a)	**09.**	(b)	**10.**	(a)	**11.**	(d)	**12.**	(b)	**13.**	(b)	**14.**	(b)
15.	(d)	**16.**	(d)	**17.**	(d)	**18.**	(b)	**19.**	(a)	**20.**	(b)		

23. $\pm\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ **OR** $\frac{\pi}{2}$ **24.** $A=\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$

25. $\because$ S.D. $=0$ therefore, the bomb may hit the fighter-jet. **27.** $-\frac{1}{\sqrt{2}}$; $2\sqrt{2}$

28. $x^2=y+\sqrt{y^2-x^2}$ **OR** $y=\frac{x^2}{4}$ **29.** 90° **30.** $\frac{1}{2}$ i.e., 50%

31. $\frac{1}{4}\log\left|x^4-9\right|+\frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|+C$ **OR** $\frac{23}{2}$ **32.** $\frac{a^2}{4}(\pi-2)$ Sq.units

33. $\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\lambda(2\hat{i}-7\hat{j}+4\hat{k})$; $\frac{x-2}{2}=\frac{y+1}{-7}=\frac{z-3}{4}$

OR $(3, 5, 9)$; $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{6}$; 7 units

34. $Z=400$ is maximum at (0, 200); $Z_{max}-Z_{min}=300$ **35.** **OR** $\frac{10\pi R^3}{3(R+5)^2}$ km^2/min

36. **(i)** $X=0, 1, 2, 3$ **(ii)** $\frac{1}{6}$ **(iii)** $\frac{2}{3}$; $\frac{1}{3}$ **OR** **(iii)** $\frac{1}{3}$

37. **(i)** $y=\frac{10-(\pi+2)x}{4}$ **(ii)** $10x-\left(2+\frac{1}{2}\pi\right)x^2$ **(iii)** $\frac{50}{\pi+4}$ m^2 **OR**

(iii) Length and Breadth of rectangular portion of the window are given by $\left(\frac{20}{\pi+4}\right)$ m and $\left(\frac{10}{\pi+4}\right)$ m respectively; Radius of semi-circular opening of window is given by $\left(\frac{10}{\pi+4}\right)$ m.

38. **(i)** $\begin{pmatrix}5 & 4 & 3\\ 4 & 3 & 5\\ 1 & 1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}11000\\ 10700\\ 2700\end{pmatrix}$ **(ii)** Amount for hockey = ₹1000, amount for cricket = ₹900 and, amount for football = ₹800.

Answers (PTS-15)

01.	(a)	**02.**	(c)	**03.**	(b)	**04.**	(d)	**05.**	(b)	**06.**	(a)	**07.**	(b)
08.	(d)	**09.**	(b)	**10.**	(c)	**11.**	(b)	**12.**	(c)	**13.**	(b)	**14.**	(d)
15.	(c)	**16.**	(c)	**17.**	(d)	**18.**	(a)	**19.**	(a)	**20.**	(a)		

21. 80° **OR** R is reflexive but not symmetric.

23. $\sqrt{42}$ Sq. units **OR** $\frac{x-1}{-2}=\frac{y-2}{1}=\frac{z-3}{0}$ **24.** $\frac{5}{3}$ **25.** $\frac{\pi}{4}$

26. $x+\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}-2\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}+C$

27.

X	0	1	2	3
P(X)	$\frac{56}{220}$	$\frac{112}{220}$	$\frac{48}{220}$	$\frac{4}{220}$

Mean = 1 **OR** $\frac{126}{295}$

28. $\frac{\pi(\pi-2)}{2}$ **OR** 4

29. $-\frac{1}{2}(5+4x-2x^2)^{3/2}+4\sqrt{2}(x-1)\sqrt{\frac{5}{2}+2x-x^2}+14\sqrt{2}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}}+C$

30. $y=\frac{k-\cos 2x}{2\cos x}$ **OR** $y\left(\log\frac{y}{x}-1\right)=x(\log x+C)$

31. Maximum value of Z is 57; minimum value of Z is 29.

33. $\left(\frac{15\pi}{2}-\frac{36}{5}-15\sin^{-1}\frac{3}{5}\right)$ Sq.units **OR** 2 Sq.units

34. 9 units; $\frac{x-5}{2}=\frac{y-4}{2}=\frac{z-0}{1}$ **OR** (1, 0, 7) **35.** $x=500,\ y=2000,\ z=3500$

36. **(i)** $(a-8)(b-12)\ cm^2$ **(ii)** $b=\frac{288+12a}{a-8}$

(iii) $A=12\left(a+32+\frac{256}{a-8}\right)$; $\frac{dA}{da}=12\left(1-\frac{256}{(a-8)^2}\right)$; $\frac{d^2A}{da^2}=12\times\frac{512}{(a-8)^3}$; $a=24$ cm.

OR **(iii)** $a=24$ cm; $b=36$ cm; $864\ cm^2$.

37. **(i)** $L=x+2y$ **(ii)** $L=x+\frac{200}{x}$ **(iii)** $x=10\sqrt{2}$ units

OR **(iii)** $y=5\sqrt{2}$ units; minimum value of $L=20\sqrt{2}$ units.

38. **(i)** $\frac{5}{9}$ **(ii)** $\frac{3}{5}$.

Answers (PTS-16)

01.	(d)	**02.**	(d)	**03.**	(c)	**04.**	(b)	**05.**	(c)	**06.**	(c)	**07.**	(b)
08.	(a)	**09.**	(b)	**10.**	(b)	**11.**	(c)	**12.**	(c)	**13.**	(b)	**14.**	(d)
15.	(d)	**16.**	(c)	**17.**	(d)	**18.**	(d)	**19.**	(d)	**20.**	(b)		

21. $2\sin^{-1}x$ **22.** $0.32\pi\ cm^2/s$ **23.** $2\hat{i}-\hat{j}+k$ **24.** **OR** $X=\begin{pmatrix}1 & -2\\ 2 & 0\end{pmatrix}$

25. $\sin^{-1}\left(\frac{e^x+2}{3}\right)+C$ **OR** $x\cos 2a-\sin 2a\log|\sin(x+a)|+C$

26. f is not onto **OR** R is equivalence relation.

27. f(x) is increasing on $x\in[0, 2]$ and decreasing on $x\in(-\infty,0]\cup[2,\infty)$

28. $y=x\log\left|\frac{x}{(x-y)^2}\right|$ **OR** $xe^x-e^x+1=\sqrt{1-y^2}$ **29.** $\mu=\frac{1}{4}$

30.

X	0	1	2
P(X)	$\frac{9}{87}$	$\frac{40}{87}$	$\frac{38}{87}$

Mean $= \frac{116}{87}$

31. **OR** $f(x)$ is not differentiable at $x=1$, $f(x)$ is differentiable at $x=2$

32. 4π Sq.units **33.** $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

34. Maximum value of z is 10. **OR** Maximum value of z is $22\frac{8}{13}$.

35. **OR** $2x^2-3x+1$

36. **(i)** $x+y+z=7000,\ x-y=0,\ 10x+16y+17z=110000$

(ii) $\begin{pmatrix}1 & 1 & 1\\ 1 & -1 & 0\\ 10 & 16 & 17\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}7000\\ 0\\ 110000\end{pmatrix}$

(iii) System of equations is consistent and, the system of equations will have unique solution as, $|A|\neq 0$. **OR** **(iii)** ₹1125/-, ₹4750/-.

37. **(i)** $-4\hat{i}$ **(ii)** $-\frac{5}{2}\hat{i}+\frac{3\sqrt{3}}{2}\hat{j}$ **(iii)** $\frac{3}{2}\hat{i}+\frac{3\sqrt{3}}{2}\hat{j},\ 6\sqrt{3}\hat{k}$ **OR** **(iii)** $3\sqrt{3}$ Sq.units.

38. **(i)** 0.039 **(ii)** $\frac{5}{13}$.

Answers (PTS-17)

01.	(b)	**02.**	(c)	**03.**	(d)	**04.**	(d)	**05.**	(c)	**06.**	(b)	**07.**	(a)
08.	(d)	**09.**	(b)	**10.**	(d)	**11.**	(a)	**12.**	(c)	**13.**	(b)	**14.**	(d)
15.	(c)	**16.**	(b)	**17.**	(d)	**18.**	(b)	**19.**	(c)	**20.**	(a)		

21. $\pi-2\sin^{-1}x$ or, $2\cos^{-1}x$ (both answers are possible) **22.** 9 cm^3/s

23. **OR** 2, –3, 0; $\vec{r}=-3\hat{i}+5\hat{j}-2\hat{k}+\lambda(2\hat{i}-3\hat{j})$ **24.** $\frac{y\sin(xy)}{\sin 2y - x\sin(xy)}$

25. $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

26. $\log|\cos x + x\sin x| + C$ **OR** $\log\left|\sin x - 1 + \sqrt{\sin^2 x - 2\sin x - 3}\right| + C$

27. ₹$\left(\frac{91}{54}\right)$ **OR** $\frac{1}{17}$ **28.** $\frac{125}{3}$

29. $y(e^x)+x^2=C$ **OR** $\frac{1}{2-e^y}=(x+1)$

30. Maximum value of Z is 495000; (30, 20) **31.** $\sqrt{2}\sin^{-1}(\sqrt{2}\sin x)-\sin^{-1}(\tan x)+C$

32. $\frac{32}{3}$ Sq.units

33. Set of all elements in A related to right angle triangle T is the 'set of all triangles'.
OR f is not onto.

34. $\dfrac{6x-17}{1}=\dfrac{6y-1}{0}=\dfrac{6z-17}{-1}$ **OR** $\vec{r}=2\hat{i}-3\hat{j}+4\hat{k}+\lambda(-\hat{i}+13\hat{j}-19\hat{k})$

35. $\begin{pmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}$

36. **(i)** $f(x)=-0.3x^2+kx+98.6$, being a polynomial function, is differentiable everywhere, hence, f is differentiable in $(0, 12)$. **(ii)** $k=3.6$
(iii) f is strictly increasing in (0, 6); f is strictly decreasing in (6, 12).
OR **(iii)** $x=6$ is a point of local maximum; $f(6)=109.4$.

37. **(i)** $\dfrac{x^2}{12\sqrt{3}}$ **(ii)** $\dfrac{1600-80x+x^2}{16}$

(iii) $x=\dfrac{120\sqrt{3}}{4+3\sqrt{3}}$ m; Length of wire used for fencing the square field $=\dfrac{160}{4+3\sqrt{3}}$ m

OR **(iii)** $A=\dfrac{400}{4+3\sqrt{3}}\ m^2$.

38. **(i)** $\dfrac{88}{1000}$ i.e., 8.8 % **(ii)** $\dfrac{9}{44}$.

Answers (PTS-18)

01.	(c)	**02.**	(d)	**03.**	(c)	**04.**	(b)	**05.**	(b)	**06.**	(b)	**07.**	(d)
08.	(b)	**09.**	(c)	**10.**	(b)	**11.**	(b)	**12.**	(c)	**13.**	(c)	**14.**	(d)
15.	(b)	**16.**	(c)	**17.**	(c)	**18.**	(d)	**19.**	(b)	**20.**	(d)		

21. $\dfrac{3\pi}{8}$ **OR** 4096; 1024 **22.** 0.002 cm/s **23.** $\dfrac{\pi}{2}$ **24.** 0

25. $\dfrac{3(3\hat{i}-5\hat{j}+4\hat{k})}{50}$ **26.** $2\sqrt{\sin x}+2\sqrt{\cos x}+C$

27. $P(\text{A wins})=\dfrac{30}{61}$, $P(\text{B wins})=\dfrac{31}{61}$ **OR** $\dfrac{196}{245}$ **28.** $2\sqrt{2}$ **OR** $\dfrac{\pi}{2}\cdot\tan^{-1}\left(\dfrac{1}{2}\right)$

29. $y=2\tan\dfrac{x}{2}-x+C$ **OR** $\sin\left(\dfrac{y}{x}\right)=\log x+C$ **30.** Minimum value of Z is 1980.

31. $I=\sin^{-1}x+\sqrt{1-x^2}+C$ **32.** $\dfrac{13}{3}$ Sq. units

34. S.D. $=\dfrac{10}{\sqrt{59}}$ units, $\cos^{-1}\left(\dfrac{13}{2\sqrt{57}}\right)$ **OR** 10 units **35.** $x=5, y=8, z=8$

36. **(i)** $\dfrac{4}{10}, \dfrac{4}{10}, \dfrac{2}{10}$ **(ii)** 1.4 **(iii)** 0.49 or 49% **OR** $\dfrac{16}{51}$.

37. **(i)** $A=\dfrac{12}{5}x\sqrt{25-x^2}$, $x\in(0, 5)$. **(ii)** $x=\dfrac{5}{\sqrt{2}}$

(iii) Length should be $5\sqrt{2}$ units and width should be $3\sqrt{2}$ units.
OR **(iii)** Length should be $5\sqrt{2}$ units and width should be $3\sqrt{2}$ units.

38. **(i)** 1 metre/hour **(ii)** 62.8 m^2/hour.

Answers (PTS-19)

01.	(c)	**02.**	(b)	**03.**	(d)	**04.**	(d)	**05.**	(b)	**06.**	(c)	**07.**	(b)
08.	(d)	**09.**	(b)	**10.**	(b)	**11.**	(b)	**12.**	(b)	**13.**	(d)	**14.**	(b)
15.	(c)	**16.**	(d)	**17.**	(b)	**18.**	(c)	**19.**	(d)	**20.**	(a)		

21. $3\sin^{-1}x$ **22.** $V=\frac{1}{384\pi^2}$ cubic units

23. $2\hat{i}+2\hat{j}$ **OR** $\pi-\cos\left(\frac{8}{21}\right)$ **24.** $-\left\{\frac{yx^{y-1}+y^x\times\log y}{x^y\times\log x+y^{x-1}\times x}\right\}$ **25.** 2

26. $\frac{1}{3}\log|x+4|+\frac{2}{3}\log|x+1|+C$ **27.** $P(B)=\frac{1}{3},\ P(B')=\frac{2}{3}$ **OR** 0.488

28. 2π **OR** $\frac{\pi}{2ab}$ **29.** $y=\frac{1}{2x^2+1}$ **OR** $\frac{1}{2}\log|y^2+x^2|-\tan^{-1}\frac{y}{x}=C$.

30. No maximum value of Z occurs. **31.** $\frac{e^x}{\log x}+C$ **32.** $\frac{13}{3}$ Sq. units

33. Yes, R is equivalence relation. **OR** $f(x)$ and $g(x)$ both are not onto.

34. **OR** $\frac{x-1}{1}=\frac{y}{-2}=\frac{z}{2}$, $\vec{r}=\hat{i}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$, $\overrightarrow{OP}=2\hat{j}-2\hat{k}$, $2\sqrt{2}$ units.

35. $x=1,\ y=2,\ z=3$ **36.** **(i)** $4-x$ **(ii)** 4 days **(iii)** 8 cm, 6 cm **OR** **(iii)** 7 days

37. **(i)** $h=\frac{2000}{\pi r^2}$ **(ii)** $A=\frac{2000}{r}+\pi r^2+\frac{4000}{\pi r}$

(iii) $r=\left[\frac{1000(\pi+2)}{\pi^2}\right]^{1/3}$, $\frac{d^2A}{dr^2}=\frac{4000}{r^3}+2\pi+\frac{8000}{\pi r^3}$ **OR** **(iii)** $(2r):h=(\pi+2):\pi$

38. **(i)** When Amrita gets success in first throw, she gets ₹5.
If she gets success in second throw, she gets ₹4.
If she gets success in third throw, she gets ₹3.
If she gets no success at all, she loses ₹3.
Clearly, values of X are 5, 4, 3, –3.
(ii) Probability distribution table is given below :

X	5	4	3	–3
P(X)	$\frac{9}{27}$	$\frac{6}{27}$	$\frac{4}{27}$	$\frac{8}{27}$

$\therefore$ Expected amount Amrita wins is, ₹$\left(\frac{19}{9}\right)$, on an average.

Answers (PTS-20)

01.	(d)	**02.**	(c)	**03.**	(c)	**04.**	(b)	**05.**	(d)	**06.**	(a)	**07.**	(d)
08.	(b)	**09.**	(a)	**10.**	(d)	**11.**	(d)	**12.**	(d)	**13.**	(b)	**14.**	(c)
15.	(b)	**16.**	(c)	**17.**	(c)	**18.**	(d)	**19.**	(d)	**20.**	(c)	**21.**	$-\frac{2\pi}{5}$

22. (2, 4) **23.** $\frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}}$ **OR** $\frac{\sqrt{34}}{2}$ units **24.** $\left(x+\frac{1}{x}\right)^x\times\left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right]$.

25. $k=1$; $\vec{r}=2\hat{i}+7\hat{j}-3\hat{k}+\lambda(-3\hat{i}+\hat{j}+2\hat{k})$; $\vec{r}=\hat{i}+3\hat{j}+6\hat{k}+\lambda(-3\hat{i}+\hat{j}-5\hat{k})$

26. $\frac{1}{3}\log|\sin 3x|+\frac{1}{2}\log|\sin 2x|+C$

27.

X	3	4	5	6	7
P(X)	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$

Mean = 5

OR $\frac{7}{11}$

28. 2 **OR** $\frac{\pi}{2}\cdot\tan^{-1}\left(\frac{1}{2}\right)$ **29.** $\sec\left(\frac{y}{x}\right)=Cxy$ **OR** $x=(\tan^{-1}y-1)+C(e^{-\tan^{-1}y})$

30. Maximum value of Z is 63 at (0, 9); minimum value of Z is 6 at (2, 0).

31. $10\log|\sin x-4|-7\log|\sin x-3|+C$ **32.** 4 Sq.units

33. R is not reflexive, R is not symmetric, R is not transitive; R is not equivalence relation.

34. $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1};\ \frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$ **35.** $x=1,\ y=1,\ z=1$

36. **(i)** $y=\frac{250}{x^2}$ **(ii)** $C(y)=₹\left(\frac{12500}{y}+400\times y^2\right)$

(iii) $y=\frac{5}{2}$ m; $C''(y)=\frac{25000}{y^3}+800$; ₹7500 **OR** **(iii)** $x=10$ m; ₹2500.

37. **(i)** $x=12.5$ **(ii)** ₹38281.25.

(iii) P(x) is strictly increasing in $x\in(0, 12.5)$; P(x) is strictly decreasing in $x\in(12.5, 20)$.

OR **(iii)** ₹37730; 15 units.

38. **(i)** $\frac{111}{121}$ i.e., 0.917 (approx.) **(ii)** 0.01089.

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Mathmission

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- CASE STUDY QUESTIONS
- PASSAGE - BASED QUESTIONS

O.P. GUPTA
INDIRA AWARD WINNER

ABOUT THE AUTHOR

O.P. GUPTA, having taught Math passionately over a decade, has devoted himself to this subject. Every book, study material or practice sheets, tests he has written, tries to teach serious math in a way that allows the students to learn Math without being afraid. His resources have helped students and teachers for a long time across the country. He has contributed in CBSE Question Bank (issued in April 2021).
Mr Gupta has been invited by many educational institutions for hosting sessions for the students of senior classes.
Being qualified as an Electronics & Communications engineer, he has pursued his graduation later on with Math from University of Delhi due to his passion towards Mathematics. He has been honored with the prestigious INDIRA AWARD by the Govt. of Delhi for excellence in education.

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